CSE 107 10-24-24

3.1 Continuous Random Variables

Random Variable:

$$X : \Omega \rightarrow \mathbb{R}$$

Defn

A random variable is said to be continuous iff there exists a function

$$f_X : \mathbb{R} \rightarrow \mathbb{R}$$

called Probability Density Function (PDF) such that

$$P(X \in S) = \int_S f_X(x)\,dx$$

for 'any' subset $S \subseteq \mathbb{R}$. In particular, if $S = [a,b]$

$$P(a \le X \le b) = \int_a^b f_X(x)\,dx$$

<u>Note</u>: $[a,b]$, $(a,b]$, $[a,b)$, (a,b) are the same, since

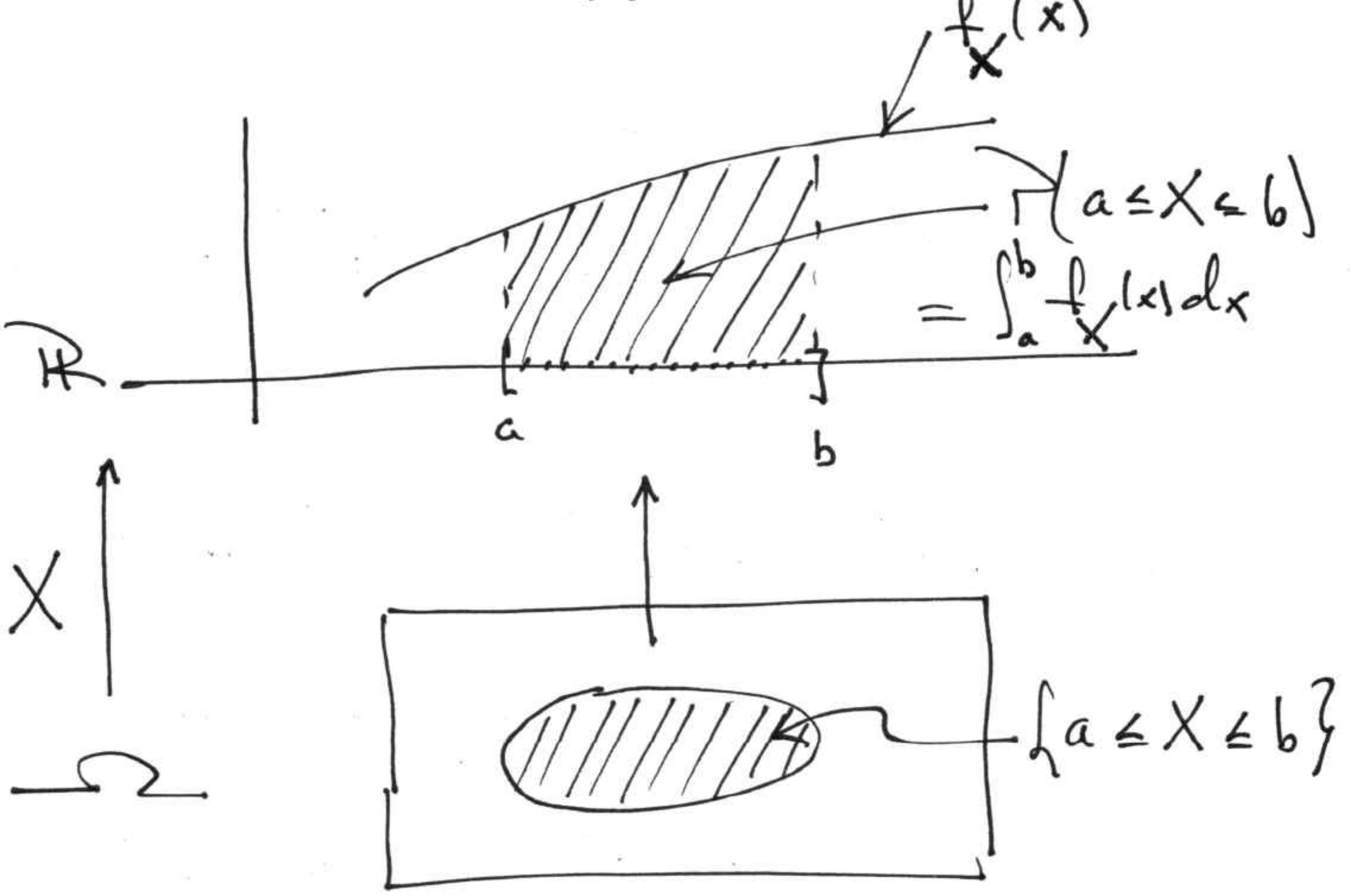
$$P(X=a) = \int_a^a f_X(x)\,dx = 0$$

Note

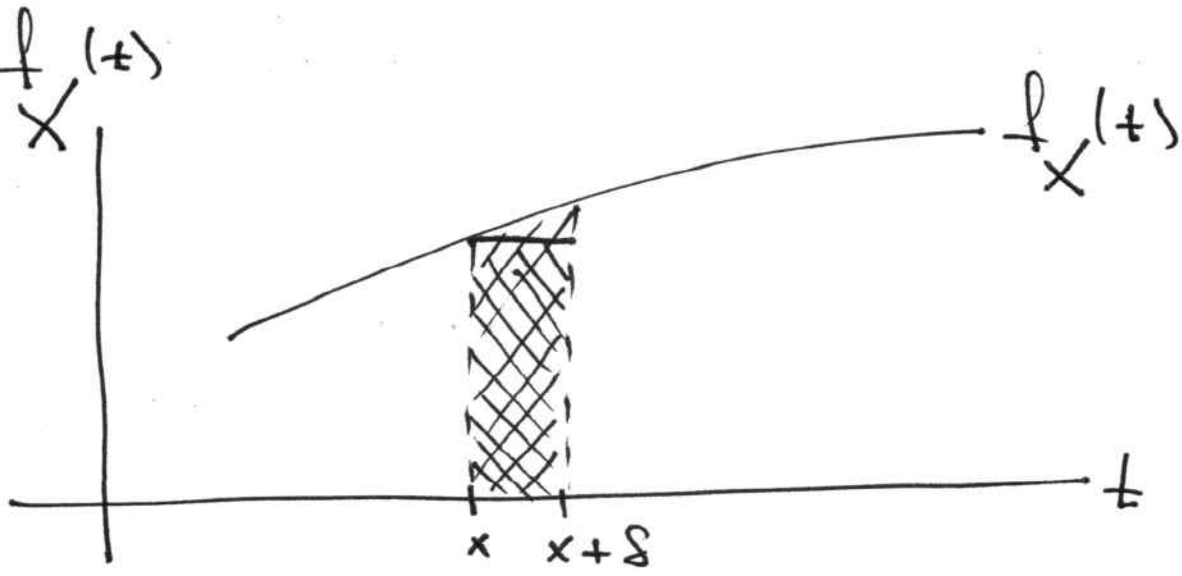
to be a valid PDF : necessarily

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

also must have $f_X(x) \geq 0$ for all $x \in \mathbb{R}$



Let $\delta > 0$ be small



$$\underbrace{P(x \le X \le x+\delta)}_{\text{Probability 'mass'}} = \int_{x}^{x+\delta} f_X(t)\,dt \approx \underbrace{f_X(x)}_{} \cdot \underbrace{\delta}_{\text{length}}$$

$$\therefore \; f_X(x) = \frac{\text{mass}}{\text{length}} = \text{density}.$$

Ex. Continuous Uniform r.v. on $[a,b]$

$$f_X(x) = \begin{cases} c & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

note: must have

$$1 = \int_{-\infty}^{\infty} f_X(x)\,dx = \int_a^b f_X(x)\,dx = \int_a^b c\,dx$$

$$= cx\Big|_a^b = c\cdot(b-a)$$

$$\therefore \quad c = \frac{1}{b-a}$$

$$\therefore \quad f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$