CSE 107
Probability and Statistics for Engineers
Information for Midterm 1

Mean and Variance Formulas:

$$E[aX + b] = aE[X] + b$$

$$E[a_1X_1 + a_2X_2 + \cdots + a_nX_n] = a_1E[X_1] + a_2E[X_2] + \cdots + a_nE[X_n]$$

$$Var(aX + b) = a^2Var(X)$$

Baye's Rule, Total Probability and Total Expectation Formulas:

Let $A_1, A_2, \dots, A_n$ be events forming a partition of Ω, let $B \subseteq \Omega$ be an event, and let $X: \Omega \to \mathbb{R}$ be a random variable on Ω.

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} \qquad \text{for each } i = 1, 2, \dots, n$$

$$P(B) = \sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)$$

$$E[X] = \sum_{i=1}^{n} P(A_i) \cdot E[X|A_i]$$

Summation Formulas:

$$\sum_{k=0}^{n-1} ar^k = a\left(\frac{1-r^n}{1-r}\right) \qquad \text{for any } r \neq 1$$

$$\sum_{k=0}^{\infty} ar^k = a\left(\frac{1}{1-r}\right) \qquad \text{for } -1 < r < 1$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \qquad \text{for any } x \in \mathbb{R}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Discrete Random Variables:

Discrete Uniform on $\{a, a+1, a+2, \dots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \dfrac{1}{b-a+1} & \text{if } a \le k \le b \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{a+b}{2} \qquad \text{Var}(X) = \frac{(b-a)(b-a+2)}{12}$$

Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1-p & \text{if } k = 0 \end{cases}$$

$$E[X] = p \qquad \text{Var}(X) = p(1-p)$$

Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{if } 0 \le k \le n \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = np \qquad \text{Var(X)} = np(1-p)$$

Geometric with parameter p

$$p_X(k) = \begin{cases} (1-p)^{k-1} \cdot p & \text{if } k = 1, 2, 3, \dots \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p} \qquad \text{Var}(X) = \frac{1-p}{p^2}$$

Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \dfrac{\lambda^k}{k!} & \text{if } k = 0, 1, 2, 3, \dots \dots \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \lambda \qquad \text{Var}(X) = \lambda$$