CSE 107
Homework Assignment 6

Section 3.3: Normal Random Variables

1. Let X be normal with mean 1 and variance 4, and let $Y = 2X + 3$.

   a. Calculate the PDF of Y.
   b. Find $P(Y \geq 0)$.

2. A signal of amplitude $s = 2$ is transmitted from a satellite and is corrupted by noise. The received signal is $X = s + W$, where the noise W is normal. When the weather is good, W has mean 0 and variance 1. When the weather is bad, W has mean 0 and variance 4. Good and bad weather are equally likely. In the absence of any weather information, determine the following.

   a. The PDF of X.
   b. The probability that X is between 1 and 3.

3. A transmitter sends the signals -1 and $+1$, with probabilities p and $1-p$, respectively. The communication channel introduces additive noise that is normal with mean 0 and variance σ^2. This noise is independent of the particular signal sent. The receiver at the other end of the channel receives a signal which is the sum of the transmitted signal and the channel noise.

   a. Let a be a constant between -1 and $+1$. The receiver interpretes its signal as -1 if it is less than a, and as $+1$ if it is greater than a. Find a formula (in terms of a, p and σ) for the probability that the receiver makes an error.
   b. Determine the probability that the receiver makes an error if $p = 2/5$, $a = 1/2$ and $\sigma^2 = 1/4$.

Section 3.4: Multiple Continuous Random Variables

4. An old modem can take anywhere from 0 to 30 seconds to establish a connection, with all times between 0 and 30 being equally likely.
   a. What is the probability that if you use this modem you will have to wait more than 15 seconds to connect?
   b. Given that you have already waited 10 seconds, what is the probability of having to wait at least 10 more seconds?

5. Consider a random variable X with PDF

$$f_X(x) = \begin{cases} \dfrac{2x}{3} & \text{if } 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   and let A be the event $A = \{X \geq 1.5\}$. Determine $E[X]$, $P(A)$ and $E[X|A]$.

Section 3.5: Conditioning

6. Alice is vacationing in Monte Carlo. The amount X of money she takes to the casino each evening is a random variable with PDF of the form

$$f_X(x) = \begin{cases} ax & \text{if } 0 \leq x \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

At the end of each night of gambling, the amount Y that she takes back to her hotel room is uniformily distributed between 0 and twice the amount she came with.

a. Find the value of a in the above formula for $f_X(x)$.
b. Write down the conditional PDF $f_{Y|X}(y|x)$.
c. Determine the joint PDF $f_{X,Y}(x,y)$.
d. Determine the marginal PDF $f_Y(y)$.
e. Find the expected value of her profit $Z = Y - X$.
f. What is the probability that on a given night, Alice makes a positive profit at the casino?