

CSE 107

Homework Assignment 4

Section 2.6: Conditioning

1. This is an extension of problem 4 from hw3, and utilizes the answers to parts a, b and c from that problem.

Alvin shops for probability books for K hours, where K is a random variable that is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops according to the conditional PMF

$$p_{N|K}(n|k) = \begin{cases} \frac{1}{k} & \text{if } n = 1, \dots, k \\ 0 & \text{otherwise} \end{cases}$$

- d) Find the conditional mean and variance of K , given that he bought at least 2 but no more than 3 books.
- e) The cost of each book is a random variable with mean \$30. What is the expected value of his total expenditure? Hint: Condition on the events $\{N = 1\}$, $\{N = 2\}$, $\{N = 3\}$ and $\{N = 4\}$ and use the total expectation theorem.

Section 2.7: Independence

2. Joe Lucky plays the lottery on any given week with probability p , independently of whether he played on any other week. Each time he plays, he has a probability q of winning, again independently of everything else. During a fixed time period of n weeks, let X be the number of weeks that he played the lottery and Y be the number of weeks that he won.
 - a. What is the probability that he played the lottery on any particular week, given that he did not win on that week?
 - b. Find the conditional PMF $p_{Y|X}(y|x)$.
 - c. Find the joint PMF $p_{X,Y}(x,y)$.

Section 3.1: Continuous Random Variables and PDFs

3. The runner-up in a road race is given a reward that depends on the difference between his time and the winner's time. He is given 10 dollars for being one minute behind, 6 dollars for being one to three minutes behind, 2 dollars for being 3 to 6 minutes behind, and nothing otherwise. Given that the difference between his time and the winner's time is uniformly distributed between 0 and 12 minutes, find the mean and variance of the reward of the runner-up.
4. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} \frac{2x}{3} & \text{if } 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and let $Y = X^2$. Calculate $E[Y]$ and $Var(Y)$.