

CSE 107

Homework Assignment 3

Section 2.4: Expectation, Mean and Variance

1. A class with 300 students consists of 250 undergraduate students and 50 graduate students. The probability that an undergraduate student gets an A is $1/3$, and the probability that a graduate student gets an A is $1/2$. Let X be the number of students who get an A. Determine the expectation $E[X]$.

Hint: define random variables

$$U_i = \begin{cases} 1 & \text{if undergraduate student } i \text{ gets an A} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } 1 \leq i \leq 250$$

and

$$G_j = \begin{cases} 1 & \text{if graduate student } j \text{ gets an A} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } 1 \leq j \leq 50$$

Write X in terms of the U_i and G_j , then use linearity of expectation.

Section 2.5: Joint PMFs of multiple random variables

2. The UCSC football team wins any one game with probability p , and loses it with probability $1 - p$. Its performance in each game is independent of its performance in other games. Let L_1 be the number of losses before its first win, and let L_2 be the number of losses after its first win and before its second win. Find the joint PMF of L_1 and L_2 .
3. A class of n students takes a test in which each student gets an A with probability p , a B with probability q , and a grade below B with probability $1 - p - q$, independently of any other student. Let X and Y be the numbers of students that getting A's and a B's, respectively. Determine the joint PMF $p_{X,Y}(x, y)$.

Section 2.6: Conditioning

4. Alvin shops for probability books for K hours, where K is a random variable that is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops according to the conditional PMF

$$p_{N|K}(n|k) = \begin{cases} \frac{1}{k} & \text{if } n = 1, \dots, k \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the joint PMF of K and N .
- (b) Find the marginal PMF of N .
- (c) Find the conditional PMF of K given that $N = 2$.