CSE 107
Homework Assignment 1

Section 1.2:

1. We roll a four-sided die once and then we roll it as many times as is necessary to obtain a different face than the one obtained in the first roll. Let the outcome of the experiment be (r_1, r_2) where r_1 and r_2 are the results of the first and the last rolls, respectively. Assume that all possible outcomes have equal probability. Find the probability that:

   (a) r_1 is even.
   (b) Both r_1 and r_2 are even.
   (c) $r_1 + r_2 < 5$.

2. You enter a special kind of chess tournament, whereby you play one game with each of three opponents, but you get to choose the order in which you play your opponents. You win the tournament if you win two games in a row. You know your probability of a win against each of the three opponents. What is your probability of winning the tournament, assuming that you choose the optimal order of playing the opponents?

3. Alice and Bob each choose at random a number between zero and two. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:

   A: The magnitude of the difference of the two numbers is greater than 1/3.
   B: At least one of the numbers is greater than 1/3.
   C: The two numbers are equal.
   D: Alice's number is greater than 1/3.

   Find the probabilities $P(A)$, $P(B)$, $P(A \cap B)$, $P(C)$, $P(D)$, $P(A \cap D)$.

Section 1.3:

4. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

   (a) Find the probability that doubles were rolled.
   (b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.
   (c) Find the probability that at least one die is a 6.
   (d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.

Section 1.4:

5. Alice and Bob have $2n + 1$ coins, each with probability of a head equal to $1/2$. Bob tosses $n + 1$ coins, while Alice tosses the remaining n coins. Show that the probability that after all the coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

6. A magnetic tape storing information in binary form has been corrupted, so it can only be read with bit errors. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Each digit is a 1 or a 0 with equal probability. Given that you read a 1, what is the probability that this is a correct reading?

Section 1.5:

7. A peculiar six-sided die has uneven faces. In particular, the faces showing 1 or 6 are 1×1.5 inches, the faces showing 2 or 5 are 1×0.4 inches, and the faces showing 3 or 4 are 0.4×1.5 inches. Assume that the probability of a particular face coming up is proportional to its area. We independently roll the die twice. What is the probability that we get doubles?

8. A parking lot consists of a single row containing n parking spaces ($n \geq 2$). Mary arrives when all spaces are free. Tom is the next person to arrive. Each person makes an equally likely choice among all available spaces at the time of arrival. Describe the sample space. Obtain P(A), the probability the parking spaces selected by Mary and Tom are at most 2 spaces apart.

9. A particular jury consists of 7 jurors. Each juror has a 0.2 chance of making the wrong decision, independently of the others. If the jury reaches a decision by majority rule, what is the probability that it will reach a wrong decision?

10. Suppose that events A, B, and C are independent. Use the definition of independence to show that A and $B \cup C$ are independent.

Section 1.6:

11. A candy factory has an endless supply of red, orange, yellow, green, blue, and violet jelly beans. The factory packages the jelly beans into jars of 100 jelly beans each. One possible color distribution, for example, is a jar of 58 red, 22 yellow, and 20 green jelly beans. As a marketing gimmick, the factory guarantees that no two jars have the same color distribution.

    (a) What is the maximum number of jars the factory can produce?
    (b) Suppose the factory sells all of the jars you counted in (a). What is the probability that a randomly purchased jar contains no violet jelly beans?