CSE 107 9/22/22

Text: B. & T

1.1 sets : covered in CSE 16.

read this

1.2 Probability models:

A Probability model consists of

(1) A set Ω called sample space.

elements $\omega \in \Omega$ are the outcomes of some experiment. A subset $A \subseteq \Omega$ is called an event.

12) A Probability Law P that assigns to an event $A \subseteq \Omega$, a real number $P(A)$, called the Probability of A.

Axioms

(i) $P(A) \geq 0$ for all $A \subseteq \Omega$

(ii) $P(\Omega) = 1$

(iii) if $A, B \subseteq \Omega$ are disjoint ($A \cap B = \phi$), then

(additivity) • $P(A \cup B) = P(A) + P(B)$

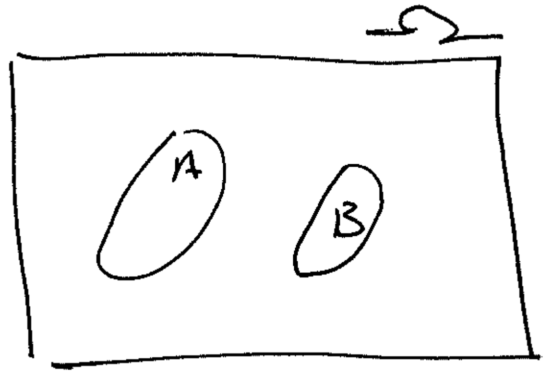
more generally, if $A_1, A_2, A_3, \ldots\ldots$ is a sequence of (Pairwise disjoint) then

(<u>countable additivity</u>)

- $P(A_1 \cup A_2 \cup A_3 \cup \ldots\ldots)$

$$= P(A_1) + P(A_2) + P(A_3) + \cdots\cdots$$

Picture:



$P(A \cup B) = P(A) + P(B)$

$A \cap B = \phi$

<u>Ex</u> flip a fair coin.

$$\Omega = \{H, T\}$$

Events: $\{H,T\}, \underbrace{\{H\}}_{\uparrow}, \underbrace{\{T\}}_{\uparrow}, \phi$

shorthand $\quad H \quad T$

note $\Omega = \{H\} \cup \{T\}$, $\{H\} \cap \{T\} = \phi$

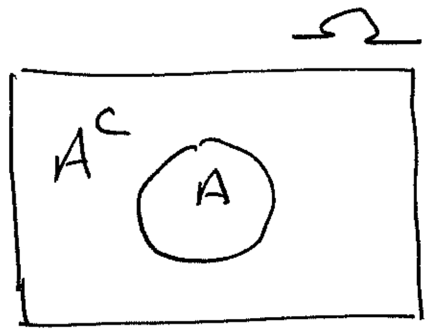
so $1 = P(\Omega) = P(H) + P(T)$.

fair means $P(H) = P(T)$, so

$\therefore$ $P(H) = P(T) = \frac{1}{2}$.

In general, if $A \subseteq \Omega$, then

$A \cap A^c = \phi$ and $A \cup A^c = \Omega$



$\therefore$ $\boxed{P(A) + P(A^c) = 1}$ hence

$P(A) \leq 1$. Thus

$$0 \leq P(A) \leq 1$$

Evidently $P(\cdot)$ is a function

$$P : \underbrace{\mathbb{P}(\Omega)}_{\substack{\text{Power set} \\ \text{of } \Omega}} \longrightarrow [0, 1]$$

let $\Omega = A$ in last equation,

$$\begin{aligned} \text{so} \quad 1 &= P(\Omega) \\ &= P(\Omega \cup \Omega^c) \\ &= P(\Omega \cup \phi) \\ &= P(\Omega) + P(\phi) \\ &= 1 + P(\phi) \end{aligned}$$

$\therefore P(\phi) = 0.$

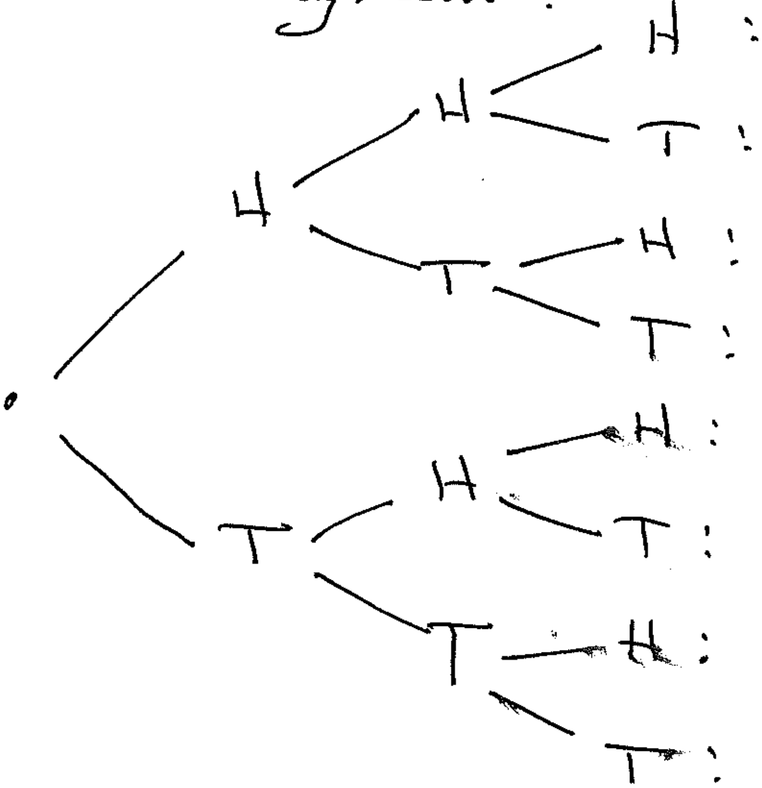
back to example

events: $\{H,T\}$, H, T, ϕ

Prob. : 1, $\frac{1}{2}$, $\frac{1}{2}$, 0

<u>Ex.</u> flip 3 fair coins

tree diagram:



<u>Outcomes</u>

$$\Omega = \left\{ \begin{array}{c} HHH \\ HHT \\ HTH \\ HTT \\ THH \\ THT \\ TTH \\ TTT \end{array} \right\}$$

fair means all outcomes are equally likely.

let $A = \{\text{exactly one head}\}$

$$= \{HTT, THT, TTH\}$$

$$= \{HTT\} \cup \{THT\} \cup \{TTH\}$$

$$P(A) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}.$$

<u>Discrete Probability law</u>

if Ω is finite, then $P(\cdot)$ is specified by Probabilities of singletons:

$P(\omega)$ for all $\omega \in \Omega$.

so if $A = \{\omega_1, \omega_2, \dots, \omega_n\}$

then

$$P(A) = P(\omega_1) + P(\omega_2) + \dots + P(\omega_n)$$

Discrete uniform law

if in addition all outcomes are equally likely, then

$$P(A) = \frac{|A|}{|\Omega|}$$