

CSE 107
Probability and Statistics for Engineers
Information for Final Exam

Expectation, Variance and Covariance Formulas:

$$E[aX + b] = aE[X] + b$$

$$E[X + Y] = E[X] + E[Y]$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$$

$$\text{Iterated Expectation Law: } E[E[X|Y]] = E[X]$$

$$\text{Total Variance Law: } \text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E[X|Y])$$

Random Variables:

Discrete Uniform on $[a, b] = \{a, a + 1, a + 2, \dots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \frac{1}{b-a+1} & \text{if } a \leq k \leq b \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{\lfloor x \rfloor - a + 1}{b - a + 1} & a \leq x < b \\ 1 & x \geq b \end{cases}$$

$$E[X] = \frac{a+b}{2} \quad \text{Var}(X) = \frac{(b-a)(b-a+1)}{12}$$

Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1-p & \text{if } k = 0 \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

$$E[X] = p \quad \text{Var}(X) = p(1-p)$$

Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{if } 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1-p)^{n-k} & 0 \leq x < n \\ 1 & x \geq n \end{cases}$$

$$E[X] = np \quad \text{Var}(X) = np(1-p)$$

Geometric with parameter p

$$p_X(k) = \begin{cases} (1-p)^{k-1} \cdot p & \text{if } k = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 - (1-p)^{\lfloor x \rfloor} & \text{if } x \geq 1 \end{cases}$$

$$E[X] = \frac{1}{p} \quad \text{Var}(X) = \frac{1-p}{p^2}$$

Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } k = 0, 1, \dots \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & \text{if } x \geq 0 \end{cases}$$

$$E[X] = \lambda \quad \text{Var}(X) = \lambda$$

Continuous Uniform on $[a, b]$ where $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$$

$$E[X] = \frac{a+b}{2} \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

Exponential with parameter λ

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

Normal with mean μ and variance σ^2

$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad F_X(x) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$E[X] = \mu \quad \text{Var}(X) = \sigma^2$$

Standard Normal ($\mu = 0$ and $\sigma = 1$)

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

$$E[X] = 0 \quad \text{Var}(X) = 1$$

Distributions Associated with Random Processes

Pascal Distribution of order k

Let Y_k be the k^{th} arrival time in a Bernoulli process with parameter p . Then $Y_k = T_1 + T_2 + \cdots + T_k$ where T_i are independent geometric random variables with parameter p .

$$p_{Y_k}(t) = \binom{t-1}{k-1} p^k (1-p)^{t-k} \quad \text{for } t = k, k+1, k+2, \dots$$

$$E[Y_k] = \frac{k}{p}$$

$$\text{Var}(Y_k) = \frac{k(1-p)}{p^2}$$

Erlang Distribution of order k

Let Y_k be the k^{th} arrival time in a Poisson process with parameter λ . Then $Y_k = T_1 + T_2 + \cdots + T_k$ where T_i are independent exponential random variables with parameter λ .

$$f_{Y_k}(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} \quad \text{for } t \in [0, \infty)$$

$$E[Y_k] = \frac{k}{\lambda}$$

$$\text{Var}(Y_k) = \frac{k}{\lambda^2}$$

Standard Normal Cumulative Distribution Function

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998