CSE 102
Homework Assignment 3

1. Define $T(n)$ by the recurrence

$$T(n) = \begin{cases} 2 & \text{if } n = 1 \\ 3T(\lfloor n/2 \rfloor) + n^2 & \text{if } n \geq 2 \end{cases}$$

   Use the substitution method to show that $T(n) = O(n^2)$.

2. Define $T(n)$ by the recurrence

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + n & \text{if } n \geq 2 \end{cases}$$

   Use the iteration method to find the exact solution to this recurrence, then determine an asymptotic solution.

3. Define $T(n)$ by the recurrence

$$T(n) = \begin{cases} 9 & \text{if } 1 \leq n < 15 \\ T(\lfloor n/2 \rfloor) + 6 & \text{if } n \geq 15 \end{cases}$$

   Use the iteration method to find the exact solution to this recurrence, then determine an asymptotic solution.

4. Define $T(n)$ by the recurrence

$$T(n) = \begin{cases} 4 & \text{if } 1 \leq n < 3 \\ T(\lfloor n/3 \rfloor) + n & \text{if } n \geq 3 \end{cases}$$

   Use iteration to find a tight asymptotic bound for $T(n)$.

5. Use the Master Theorem to find tight asymptotic bounds for the recurrences in problems 3 and 4 above.

6. Use the Master Theorem to find tight asymptotic bounds on the following recurrences.

   a. $T(n) = 3T(2n/3) + n^3$
   b. $T(n) = 2T(n/3) + \sqrt{n}$
   c. $T(n) = 5T(n/4) + n^{\lg \sqrt{5}}$
   d. $T(n) = 3T(2n/5) + n \log n$
   e. $S(n) = aS(n/4) + n^2$ (your answer will depend on the parameter a.)

7. Assume the correctness of the algorithm $\text{Partition}(A, p, r)$ on page 171 of the text. Use induction to prove the correctness of $\text{Quicksort}(A, p, r)$ on page 171.