CSE 102
Introduction to Analysis of Algorithms
Review Problems From CMPS 101

1. Determine whether the following statements are true or false. For each true statement, give a proof. For each false stament, give a counterexample.

   a. $(\log(n))^{100} = \omega(n^{0.01})$.

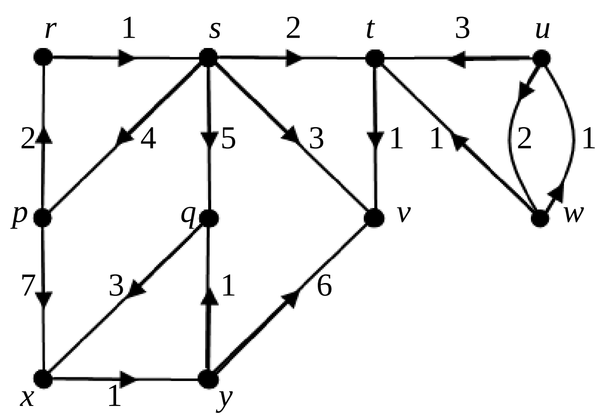
   b. If $f(n) = \Theta(g(n))$, then $L = \lim_{n\to\infty}(f(n)/g(n))$ exists and $0 < L < \infty$.

   c. $o(g(n)) \cap \Omega(g(n)) = \emptyset$ for any function $g(n)$.

   d. If $f(n) = O(g(n))$, then $2^{f(n)} = O(2^{g(n)})$.

2. Determine a tight asymptotic bound for the solution to the recurrence $T(n) = 9T(n/2) + n^{3.16}$.

3. Trace execution of Dijkstra's algorithm on the weighted digraph pictured below, taking vertex r as the source.



4. Consider the following algorithm which does nothing but waste time:

```
WasteTime(n)   (pre: n ≥ 1)
if n > 1
    for i = 1 to n^3
        waste 2 units of time
    for i = 7 to 7
        WasteTime(⌈n/2⌉)
    waste 3 units of time
```

   a. Write a recurrence formula which gives the amount of time $T(n)$ wasted by this algorithm.
   b. Show that when n is an exact power of 2, the solution to this recurrence relation is given by $T(n) = 16n^3 - \frac{1}{2} - \frac{31}{2} n^{\lg(7)}$, and hence $T(n) = \Theta(n^3)$.

5. Prove that all trees on n vertices have $n-1$ edges. Do this by (a) induction on the number of vertices, and (b) by induction on the number of edges.

6. Use Induction to show that $\forall\, n \geq 1$: $T(n) \leq 12n$, and hence $T(n) = O(n)$, where $T(n)$ is defined by the recurrence formula

$$T(n) = \begin{cases} 1 & 1 \leq n < 3 \\ 2T(\lfloor n/3 \rfloor) + 4n & n \geq 3 \end{cases}$$

7. Determine a solution to the following recurrence

$$T(n) = \begin{cases} 7 & 1 \leq n < 3 \\ 2T(\lfloor n/3 \rfloor) + 5 & n \geq 3 \end{cases}$$

<u>Problems 8 and 9 refer to the following graphs:</u>

Figure 1:

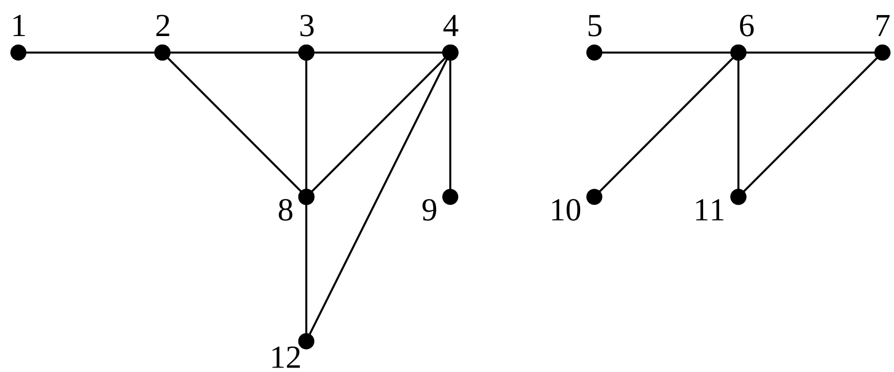


Figure 2:

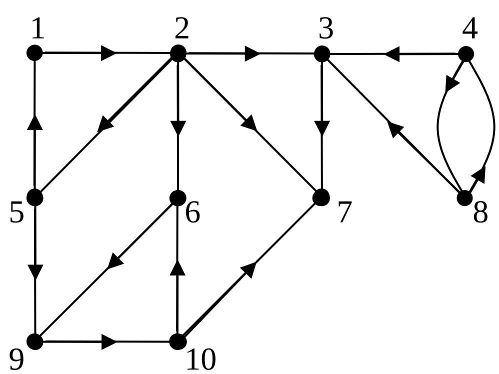
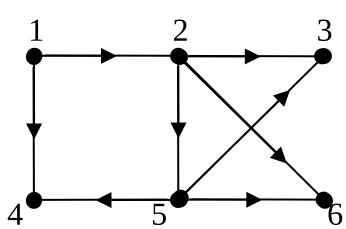


Figure 3:



8. Trace Breadth First Search on the following graphs.
   a. The graph in figure 1, with 1 as the source.
   b. The directed graph in figure 2 with 1 as source.

9. Trace Depth First Search on the following graphs.
   a. The graph in figure 1.
   b. The graph in figure 2.
   c. The graph in figure 3. Determine a topological sort of the vertices.
   d. The transpose of the graph in figure 2. Determine the strongly connected components of the graph in figure 2, and draw its component graph. Also determine a topological sort of the strong components.

10. Let T be a binary tree. Let $n(T)$ denote the number of nodes in T and $h(T)$ denote the height of T. Show that $h(T) \geq \lfloor \lg(n(T)) \rfloor$.

11. Let G be a weighted connected graph (undirected) with distinct edge weights. Show that G contains a *unique* minimum weight spanning tree.

12. The following weighted graph contains three minimum weight spanning trees. Run Kruskal's algorithm to find one, then find the other two by inspection.