CSE 102
Final Review Problems

Be sure to review all prior homework assignments, midterm exams, and their solutions. Review all examples covered in class.

1. Suppose $T(n)$ satisfies the recurrence $T(n) = 3T(n/4) + F(n)$, where $F(n)$ itself satisfies the recurrence $F(n) = 5F(n/9) + n^{3/4}$. Find a tight asymptotic bound for $T(n)$. Be sure to fully justify each use of the Master Theorem. (Hint: $\log_9(5) < 3/4 < \log_4(3)$.)

2. Suppose we are given 4 gold bars (labeled 1, 2, 3, 4), one of which *may* be counterfeit: gold-plated tin (lighter than gold) or gold-plated lead (heavier than gold). Again the problem is to determine which bar, if any, is counterfeit and what it is made of. The only tool at your disposal is a balance scale, each use of which produces one of three outcomes: tilt left, balance, or tilt right.

    a. Use a decision tree argument to prove that at least 2 weighings must be performed (in worst case) by any algorithm that solves this problem. Carefully enumerate the set of possible verdicts.
    b. Determine an algorithm that solves this problem using 3 weighings (in worst case). Express your algorithm as a decision tree.
    c. Find an adversary argument that proves 3 weighings are necessary (in worst case), and therefore the algorithm you found in (b) is best possible. (Hint: study the adversary argument for the min-max problem discussed in class to gain some insight into this problem. Further hint: put some marks on the 4 bars and design an adversary strategy that, on each weighing, removes the fewest possible marks, then show that if the balance scale is only used 2 times, not enough marks will be removed.)

3. Recall the coin changing problem again. Given denominations $d = (d_1, d_2, \ldots, d_n)$ and an amount N, determine the number of coins in each denomination necessary to disburse N units using the fewest possible coins. Assume that there is an unlimited supply of coins in each denomination. Prove that the greedy strategy works for any amount N with the coin system $d = (1, 5, 10, 25)$.

4. **Scheduling to Minimize Average Completion Time:** (This is problem 16-2a on page 402 of CLRS.) Suppose you are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of tasks, where task a_i requires p_i units of processing time to complete, once it has started. You have one computer on which to run these tasks, and the computer can run only one task at a time. Let c_i be the *completion time* of task a_i, that is, the time at which task a_i completes processing. Your goal is to minimize the average completion time, that is to minimize the quantity $(1/n)\sum_{i=1}^{n} c_i$. For example, suppose there are two tasks, a_1 and a_2, with $p_1 = 3$ and $p_2 = 5$, and consider the schedule in which a_2 runs first, followed by a_1. Then $c_2 = 5$, $c_1 = 8$, and the average completion time is $(5 + 8)/2 = 6.5$.

    Give an algorithm that schedules the tasks so as to minimize the average completion time. Each task must run non-preemptively, that is, once task a_i is started, it must run continuously for p_i units of time. Prove that your algorithm minimizes the average completion time, and state the running time of your algorithm.

5. Let $B = b_1 b_2 \dots b_n$ be a bit string of length n. Consider the following problem: determine whether or not B contains 3 consecutive 1's, i.e. whether B contains the substring "111". Consider algorithms that solve this problem whose only allowable operation is to peek at a bit.

   a. Suppose $n = 4$. Obviously 4 peeks are sufficient. Give an adversary argument showing that in general, 4 peeks are also necessary.
   b. Suppose $n \geq 5$. Give an adversary argument showing that $4 \cdot \lfloor n/5 \rfloor$ peeks are necessary. (Hint: divide B into $\lfloor n/5 \rfloor$ 4-bit blocks separated by 1-bit gaps between them. Thus bits 1-4 form the first block, and bit 5 is the first gap. Bits 6-9 form the next block and bit 10 is the next gap, etc.. Any leftover bits form a separate block. Now run the adversary from part (a) on each of the 4-bit blocks.)