CSE 102
Homework Assignment 1

1. (Problem 3.1-1) Let $f(n)$ and $g(n)$ asymptotically positive functions. Prove that $f(n) + g(n) = \Theta\big(\max\big(f(n),\ g(n)\big)\big)$.

2. Prove or disprove: If $f(n) = \Theta\big(g(n)\big)$, then $f(n)^2 = \Theta(g(n)^2)$.

3. Prove or disprove: If $f(n) = \Theta\big(g(n)\big)$, then $2^{f(n)} = \Theta\big(2^{g(n)}\big)$.

4. Let $f(n)$ and $g(n)$ be asymptotically positive functions, and assume that $\lim_{n\to\infty} g(n) = \infty$. Prove that if $f(n) = \Theta\big(g(n)\big)$, then $\ln\big(f(n)\big) = \Theta\big(\ln\big(g(n)\big)\big)$.

5. (Problem 3.2-8) Show that if $f(n) \ln f(n) = \Theta(n)$, then $f(n) = \Theta(n/\ln n)$. Hint: use the result of the preceding problem.

6. Consider the statement: $f(cn) = \Theta\big(f(n)\big)$.
   a. Determine a function $f(n)$ and a constant $c > 0$ for which the statement is false.
   b. Determine a function $f(n)$ for which the statement is true for all $c > 0$.

7. Determine the asymptotic order of the expression $\sum_{i=1}^{n} a^i$ where $a > 0$ is a constant, i.e. find a simple function $g(n)$ such that the expression is in the class $\Theta\big(g(n)\big)$. (Hint: consider the cases $a = 1$, $a > 1$, and $0 < a < 1$ separately.)

8. Use induction to prove that $\sum_{k=1}^{n} k^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$ for all $n \geq 1$.