cse 102 9/22/22

Defn

let $g(n)$ be a function. The set

$$O(g(n)) = \{ f(n) \mid \exists c>0, \exists n_0>0, \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n) \}$$

i.e. $f(n) \in O(g(n))$ iff

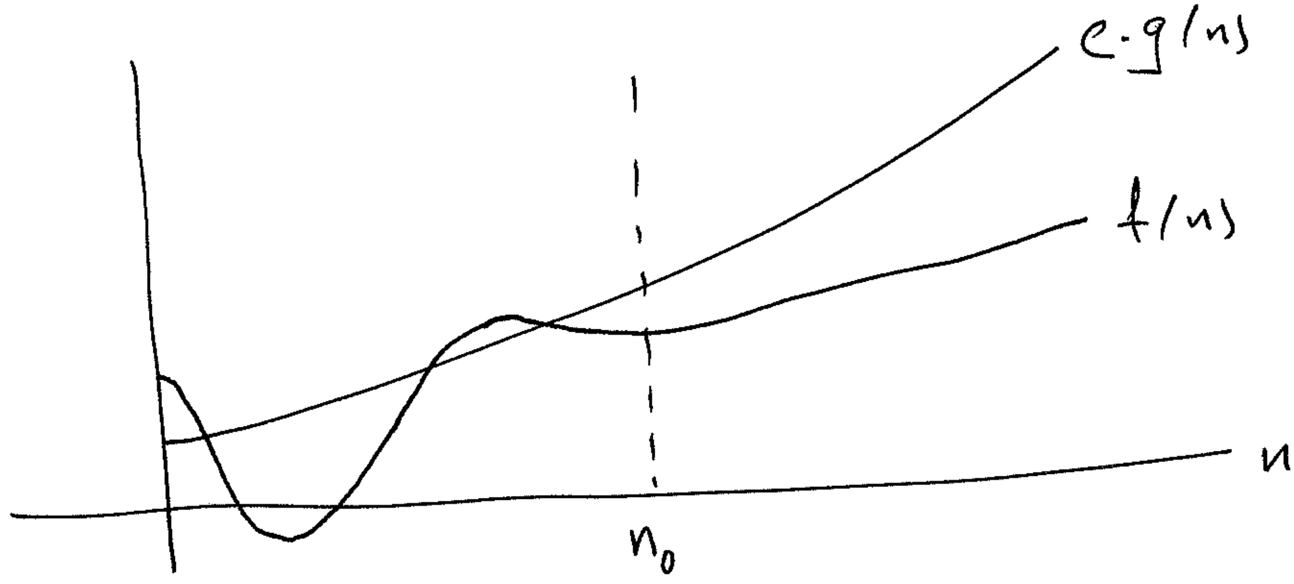
there exist positive c, n_0 such that for all $n \geq n_0$, we have

$$0 \leq f(n) \leq c \cdot g(n)$$

<u>notation</u>: $f(n) = O(g(n))$

<u>means</u>: $f(n)$ grows at most as fast as $g(n)$.

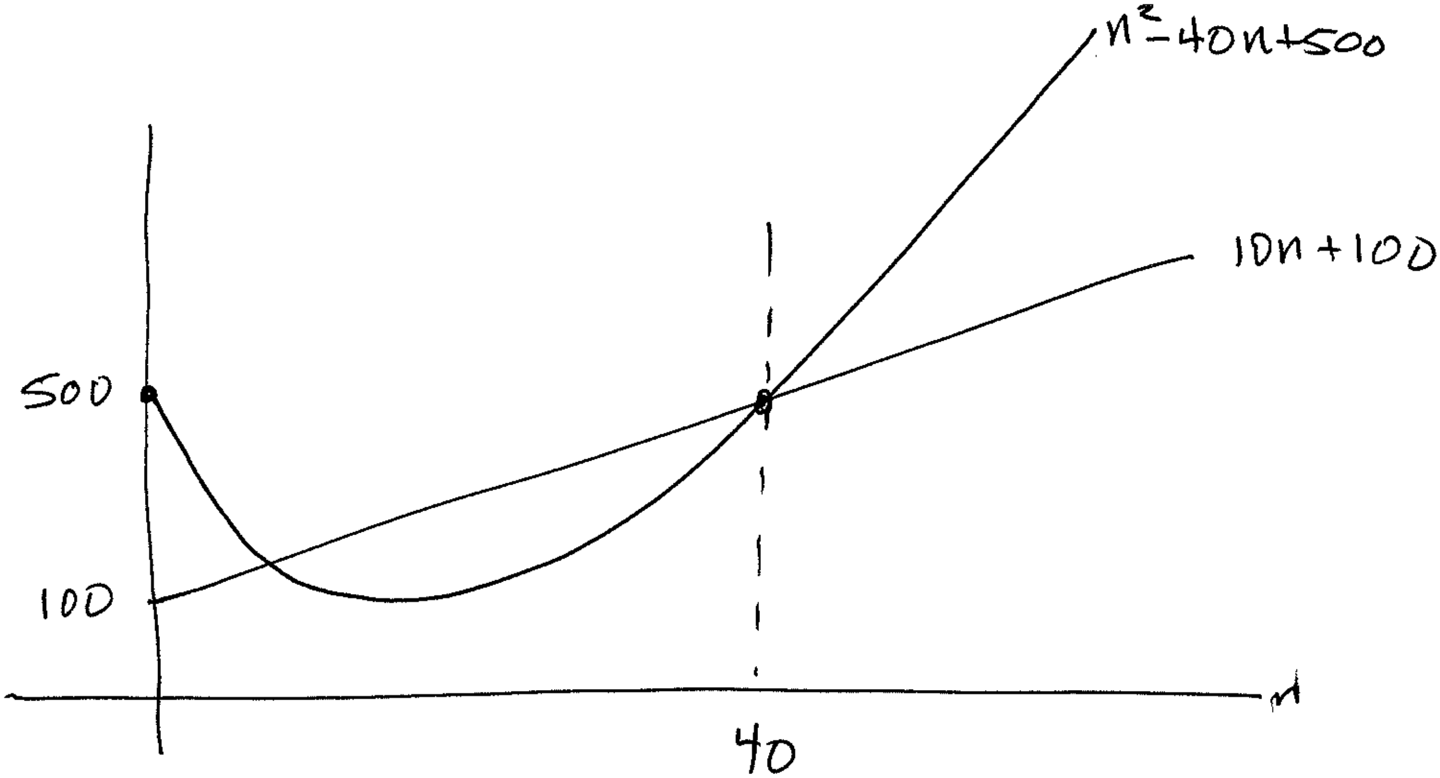
Picture:



We say: $g(n)$ is an asymptotic upper bound for $f(n)$

Ex. $10n + 100 = O(n^2 - 40n + 500)$

why? because $0 \le 10n + 100 \le 1(n^2 - 40n + 500)$ for all $n \ge 40$. so $n_0 = 40$ and $c = 1$.



- In fact $an+b = O(cn^2+dn+e)$ for any a, b, c, d, e with $a>0$, $c>0$.

- If $P(n)$, $q(n)$ are polynomials with $\deg(P(n)) \le \deg(q(n))$, then

$$P(n) = O(q(n)).$$

<u>Defn</u>

let $g(n)$ be a function.

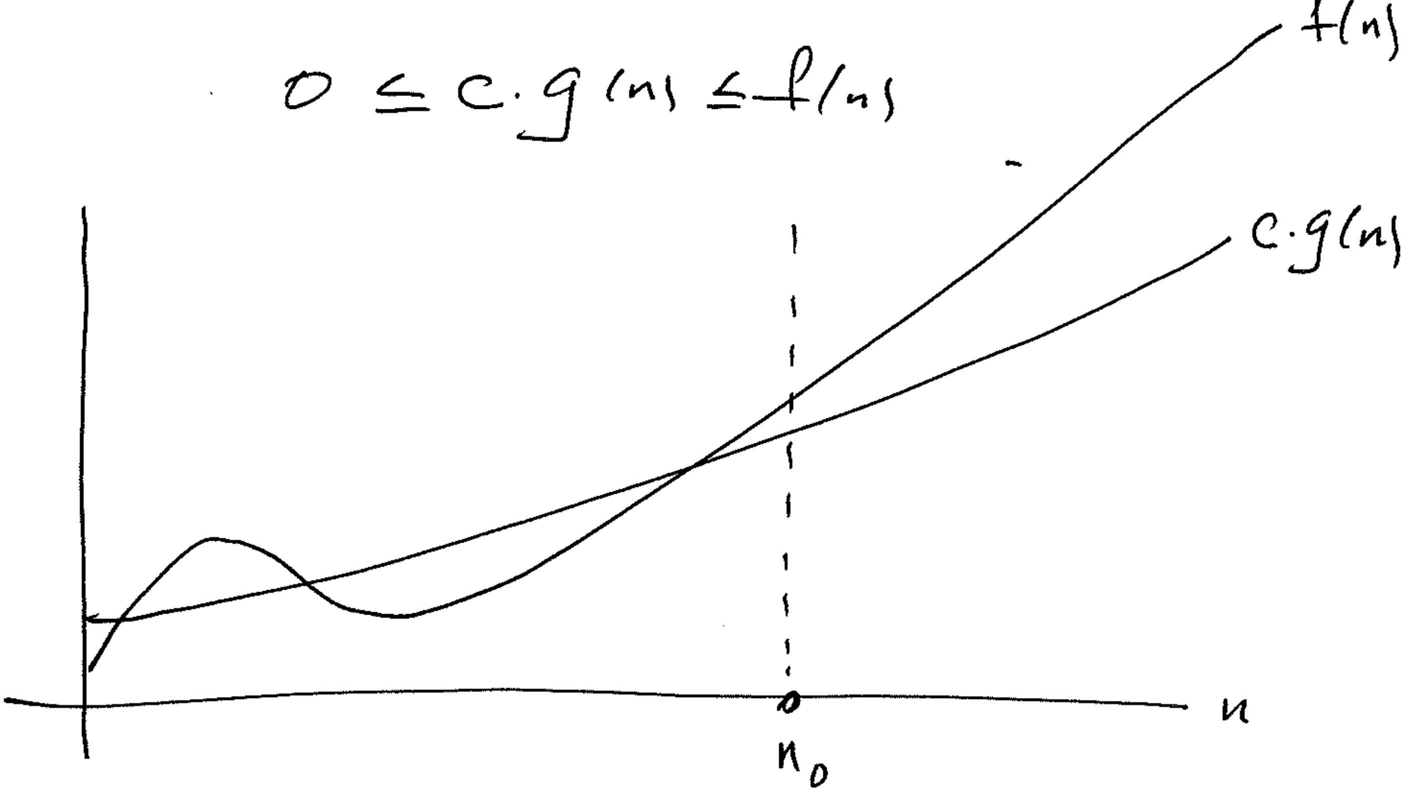
$$\Omega(g(n)) = \{ f(n) \mid \exists c>0, \exists n_0>0, \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n) \}$$

i.e. $f(n) = \Omega(g(n))$ iff

there exist positive c, n_0 s.t.

for all $n \geq n_0$, we have

$$0 \leq c \cdot g(n) \leq f(n)$$



We say $g(n)$ is an asymptotic lower bound for $f(n)$.

meaning: $f(n)$ grows no slower than $g(n)$.

Theorem

$f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

$\Longrightarrow$

$\Longleftarrow$ exercise.

Proof of $\Longrightarrow$:

If $f(n) = O(g(n))$ then there exist pos. c_1, n_1 s.t. $0 \le f(n) \le c_1 g(n)$ for all $n \ge n_1$. We must show that there exist positive c_2, n_2 s.t.

$0 \le c_2 f(n) \le g(n)$ for all $n \ge n_2$.

Let $c_2 = \frac{1}{c_1}$ and $n_2 = n_1$. Then

if $n \ge n_2$, we have $n \ge n_1$, hence

$0 \le f(n) \le c_1 g(n)$, therefore

$0 \le \frac{1}{c_1} f(n) \le g(n)$, whence we have

$0 \le c_2 \cdot f(n) \le g(n)$, as required. ∎

<u>note</u>:

- we say $f(n)$ is <u>asymptotically non-negative</u> iff there exists $n_0 > 0$ s.t. for all $n \ge n_0$, we have $f(n) \ge 0$.
- we say $f(n)$ is <u>asymptotically positive</u> iff there exists $n_0 > 0$ s.t. for all $n \ge n_0$, we have $f(n) > 0$.