CSE 102 12-1-22

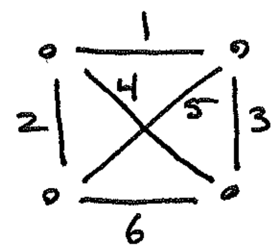
- Final: Wed. 12-7-22, 7:30-9:30 PM
- SETs: Due Sun 12-4-22 at 11:59 PM.

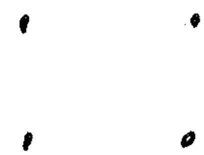
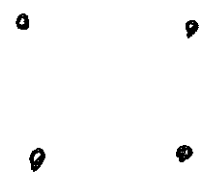
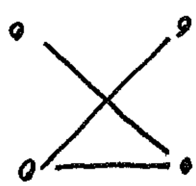
Ex Graph connectivity: $|V(G)| = n$.

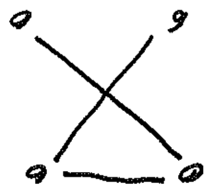
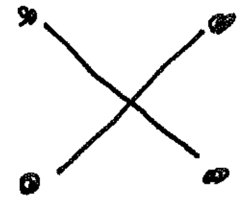
- 1st adversary L.B. $\lfloor \frac{n}{2} \rfloor \lceil \frac{n}{2} \rceil \sim \frac{n^2}{4}$
- 2nd adversary L.B. $\binom{n}{2} = \frac{n(n-1)}{2} \sim \frac{n^2}{2}$

# Illustrate Proof:

**Ex.** $n = 4$ $\quad A = E(K_4)$, $B = \phi$




| edge probe | A | B | answer |
|---|---|---|---|
| 1 |  |  | no |
| 2 |  |  | no |
| 3 |  |  | no |
| 4 |  |  | yes |

| edge | A | B | answer |
|---|---|---|---|
| 5 |  |  | yes |

## Exercise

Show that at least $\binom{n}{2}$ "adjacency" questions are necessary to determine a graph $G$ on $n$ vertices is acyclic.

## Hint:

Answer yes to any edge probe unless that answer would prove the existence of a cycle.

## Exercise

Let $b = x_1 x_2 x_3 x_4 x_5$ be a bit string of len. 5, i.e. $x_i \in \{0,1\}$

Problem: Determine whether $b$ contains substring "111", i.e. does $b$ contain 3 consecutive 1s?

Probes: Peek at a bit.

(a) design an algorithm that only does 4 peeks (in worst case).

Present algorithm as a decision tree.

(b) give an adversary argument showing that in general 4 peeks are necessary.

Ex.

we saw we can determin (min, max) in an array A

| 1 | 2 | 3 | 4 | ... | n |
|---|---|---|---|---|---|
| | | | | ... | |

by doing $\lceil \frac{3n}{2} \rceil - 2$ comparisons.

can we do better? NO.

will show by adversary argument that $\lceil \frac{3n}{2} \rceil - 2$ is a lower bound for this problem.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| A | ± | ± | ± | ± | ± | ± | ± | ± | ± | ± |

initially: $2n$ marks

| | | | |
|---|---|---|---|
| ± | candidate | for | both max, min |
| + | " | " | max only |
| − | " | " | min only |
| N | " | " | neither max nor min |