CSE 102 10-20-22

Ex.

$$T(n) = T\left(\lfloor \tfrac{n}{2} \rfloor\right) + 2T\left(\lceil \tfrac{n}{2} \rceil\right) + \log(n!)$$

Simplify:

$$T(n) = 3T\left(\tfrac{n}{2}\right) + n\log n$$

Compare $n\log n$ to $n^{\log_2 3}$

let $\varepsilon = \dfrac{\log_2(3)-1}{2} > 0$

$$2\varepsilon = \log_2 3 - 1$$

$$\varepsilon + 1 = \log_2 3 - \varepsilon$$

$$\frac{n\log n}{n^{\log_2 3 - \varepsilon}} = \frac{n\log n}{n^{1+\varepsilon}} = \frac{\cancel{n}\log n}{\cancel{n}\cdot n^{\varepsilon}} \longrightarrow 0 \quad \text{as } n \to \infty$$

$\therefore$

$$n \log n = o\left(n^{\log_2 3 - \varepsilon}\right) \subseteq O\left(n^{\log_2 3 - \varepsilon}\right)$$

$\therefore$ by case 1:

$$T(n) = \Theta\left(n^{\log_2 3}\right)$$

Ex.

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

Comp: $\frac{n}{\log n}$ to $n^{\log_2 2} = n$

should be ever 1, but no poly factor.

claim: $\frac{n}{\log n} \neq O\left(n^{\frac{1}{\log_2 2 - \varepsilon}}\right)$ for any $\varepsilon > 0$

check

$$\frac{n/\log n}{n^{1-\varepsilon}} = \frac{n/\log n}{n/n^{\varepsilon}} = \frac{n^{\varepsilon}}{\log n} \longrightarrow \infty$$

$\therefore \frac{n}{\log n} = \omega\left(n^{1-\varepsilon}\right)$, and $\omega() \cap O() = \phi$.

Exercise

find an example where

$$f(n) = \Omega\left(n^{\log_b(a)+\varepsilon}\right)$$

for some $\varepsilon > 0$, but the regularity cond. fails.

Divide and Conquer