

CSE 102 10-20-22

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Ex.

$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 2T\left(\left\lceil \frac{n}{2} \right\rceil\right) + \log(n!)$$

Simplify:

$$T(n) = 3T\left(\frac{n}{2}\right) + n \log n$$

Compare $n \log n$ to $n^{\log_2 3}$

$$\text{let } \varepsilon = \frac{\log_2(3) - 1}{2} > 0$$

$$2\varepsilon = \log_2 3 - 1$$

$$\varepsilon + 1 = \log_2 3 - \varepsilon$$

$$\frac{n \log n}{n^{\log_2 3 - \varepsilon}} = \frac{n \log n}{n^{1 + \varepsilon}} = \frac{\log n}{n \cdot n^{\varepsilon}} \rightarrow 0 \text{ as } n \rightarrow \infty$$

∴

$$n \log n = o(n^{\log_2 3 - \epsilon}) \subseteq O(n^{\log_2 3 - \epsilon})$$

∴ by case 1:

$$T(n) = \Theta(n^{\log_2 3})$$

Ex.

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

com: $\frac{n}{\log n}$ to $n^{\log_2 2} = n$

should be case 1, but no poly factor.

claim: $\frac{n}{\log n} \neq O\left(n^{\frac{1}{\log_2 2} - \varepsilon}\right)$ for
any $\varepsilon > 0$

check

$$\frac{n/\log n}{n^{1-\varepsilon}} = \frac{n/\log n}{n/n^\varepsilon} = \frac{n^\varepsilon}{\log n} \rightarrow \infty$$

$$\therefore \frac{n}{\log n} = \omega(n^{1-\varepsilon}), \text{ and } \omega(\cdot) \cap O(\cdot) = \emptyset$$

Exercise

find an example where

$$f(n) = \Omega(n^{\log_b(a) + \varepsilon})$$

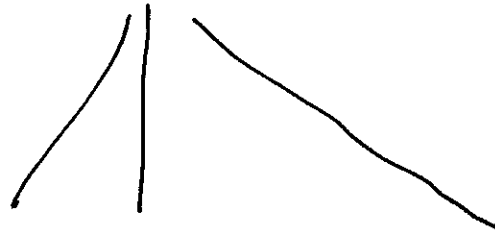
for some $\varepsilon > 0$, but the regularity
cond. fails.

Divide and Conquer

Prob. instance

I

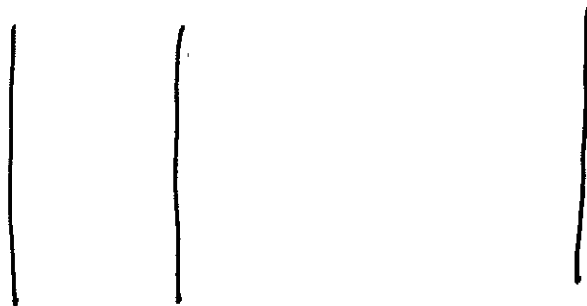
divide



Sub-instances

$I_1, I_2, \dots, I_k$

solve



Sub-instance
solutions

$S_1, S_2, \dots, S_k$

combine

solution

