

CS2 101 3-5-26

11

Part: ext. 1 last day

RBTs:

note:

$bh(x) = 0$ iff $height(x) = 0$

iff x is a leaf (nil).

Theorem

A RBT with n internal
(i.e. non-nil) nodes and height
 h satisfies

$$h \leq 2 \lg(n+1)$$

Remarks

(1) any binary tree satisfies

$h \geq \lfloor \lg n \rfloor$, so a RBT T satisfies

$$\lfloor \lg n \rfloor \leq \text{height}(T) \leq 2 \lg(n+1)$$

$\therefore$ all queries, insertions, deletions

run time

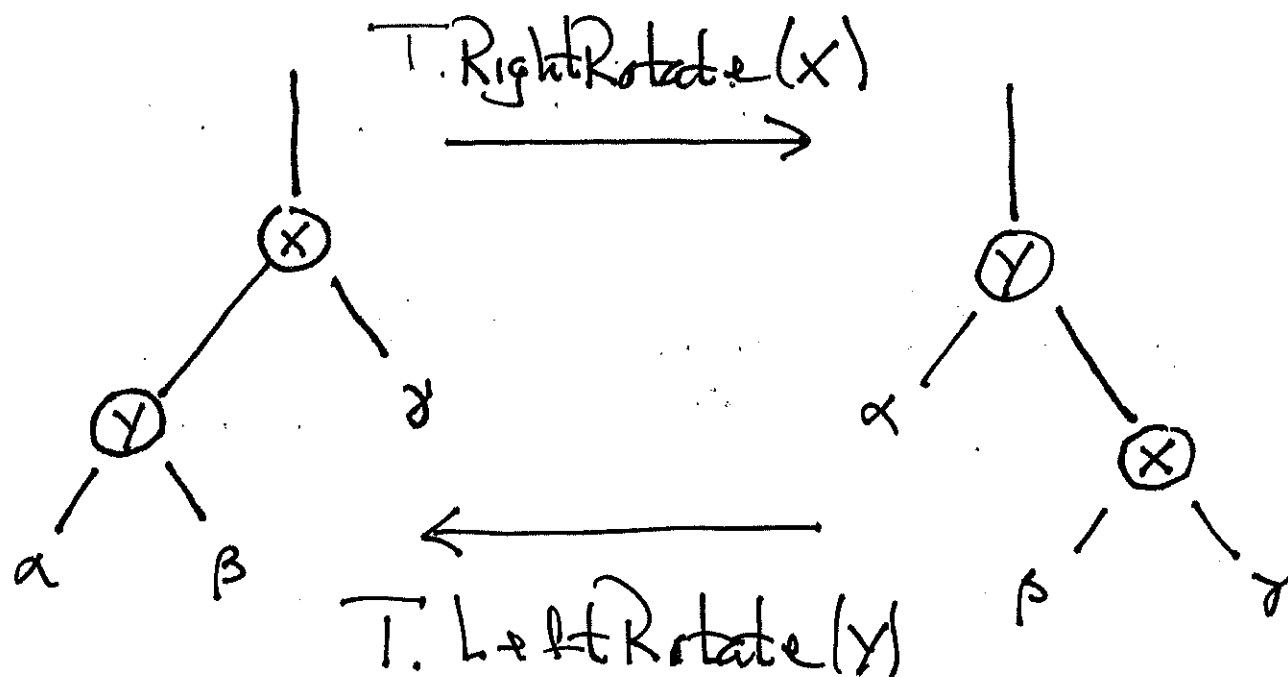
$$O(\log n)$$

(2) note $\ln(n) = o(n)$ ✓ since

$$\lim_{n \rightarrow \infty} \frac{\ln(n)}{n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{1} = 0$$

13.2 Rotations

Picture:



Summary:

- $\beta \begin{cases} x.left \leftarrow y.right \\ y.right.parent \leftarrow x \end{cases}$
- $Parent \begin{cases} y.parent \leftarrow x.parent \\ x.parent.(left \text{ or } right) \leftarrow y \end{cases}$

• $x, y \left\{ \begin{array}{l} y.\text{right} \leftarrow x \\ x.\text{Parent} \leftarrow y \end{array} \right.$

Runtime : $\Theta(1)$

note : Rotations Preserve

$$\text{keys}(\alpha) \leq y.\text{key} \leq \text{keys}(\beta) \leq x.\text{key} \leq \text{keys}(\gamma)$$

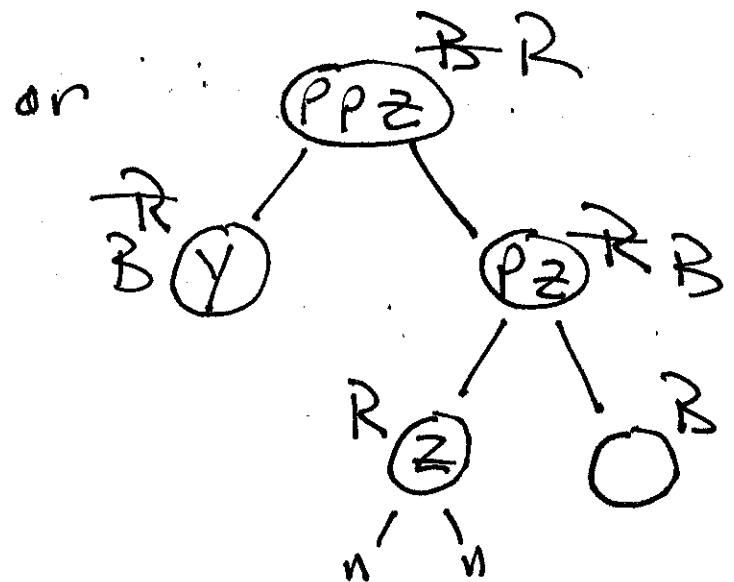
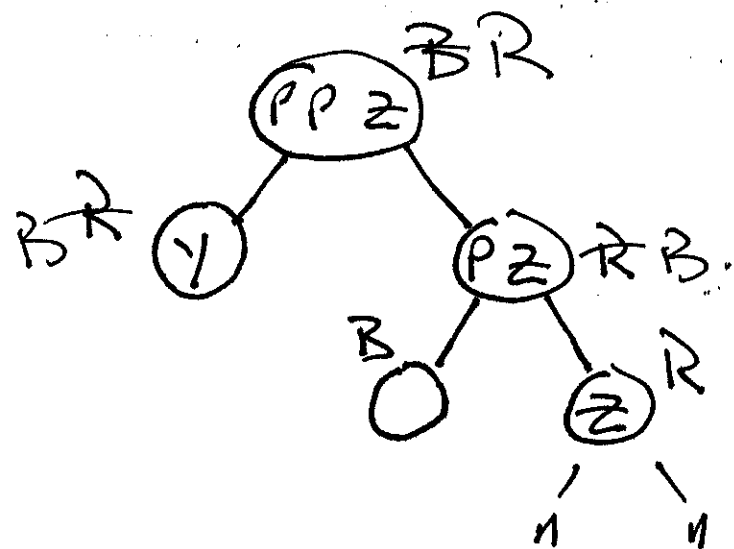
and so Preserve BST Properties.

13.3 Insertion in RBT

Cases: 4, 5, 6

- z .Parent is right child of z .Parent.Parent.
- $y = z$.Parent.Parent.left

Case 4! y is Red.

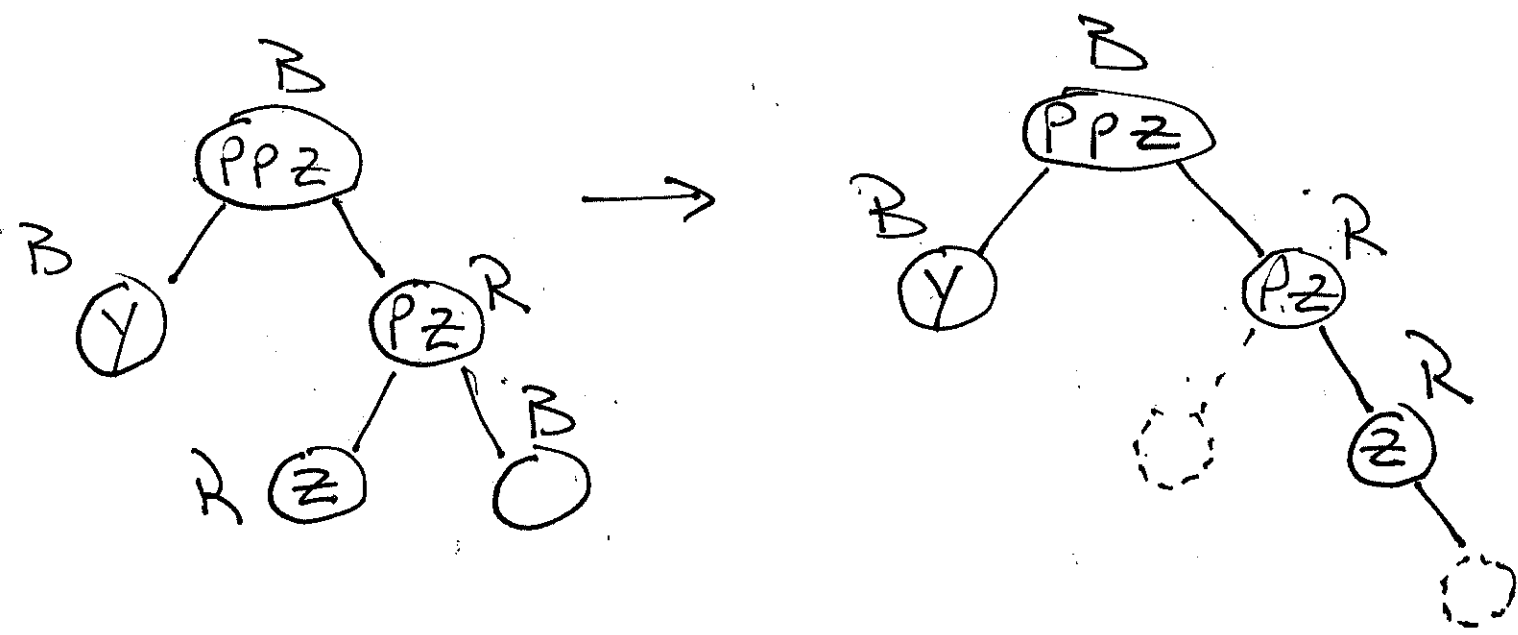


fix colors amongst y, pz, ppz ,
 then let z climb up tree
 to ppz

Case 5: y is Black, $z == z.Parent.left$

convert to Case 6:

let $z = z.Parent$, right rotate
 about z



Case 6: y is Black, $z == z.\text{parent}.\text{right}$

color Pz Black, PPz Red,
then left rotate about PPz

