CSE 101 2-3-26

Note:

- $f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$

Ex. $\ln(n) = o(n)$ ✓

Proof

$$\lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$$

Note

$$\frac{f(n)}{g(n)} \leq B_1 \quad \text{iff} \quad \frac{g(n)}{f(n)} \geq B_2 = \frac{1}{B_1}$$

so

$$f(n) = O(g(n)) \quad \text{iff} \quad g(n) = \Omega(f(n)).$$

Analogy

$$x \leq y \quad \text{iff} \quad y \geq x$$

Note:

$$f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(g(n))$$

Exercises

(3)

$$\lim_{n\to\infty}\left(\frac{n^\alpha}{n^\beta}\right) = \lim_{n\to\infty}\left(n^{\alpha-\beta}\right) = \begin{cases} 0 & \text{if } \alpha<\beta \\ 1 & \text{if } \alpha=\beta \\ \infty & \text{if } \alpha>\beta \end{cases}$$

$$\therefore \quad n^\alpha = \begin{cases} o(n^\beta) & \text{if } \alpha<\beta \\ \Theta(n^\beta) & \text{if } \alpha=\beta \\ \omega(n^\beta) & \text{if } \alpha>\beta \end{cases}$$

(4) $\lim_{n\to\infty}\left(\frac{a^n}{b^n}\right) = \lim_{n\to\infty}\left(\frac{a}{b}\right)^n = \begin{cases} 0 & \text{if } a<b \\ 1 & \text{if } a=b \\ \infty & \text{if } a>b \end{cases}$

$$\therefore\ a^n = \begin{cases} o(b^n) & a<b \\ \Theta(b^n) & a=b \\ \omega(b^n) & a>b \end{cases}$$

(5) Hint: $\log_b(n) = \frac{\ln(n)}{\ln(b)}$

so $\log_b(n) = \text{const} \cdot \log_a(n)$.

$\therefore\ \log_b(n) = \Theta(\log_a(n))$.

<u>Notation</u>

$$f(n) + \underbrace{\Theta(n^2)}_{\uparrow} = g(n)$$

Some fcn that grows like n^2

Ex.

SelectionSort(A)

```
n = length(A)
for i = 1 to n-1
    imin = i
    for j = i+1 to n
        if A[j] < A[imin]
            imin = j
    A[i] <-> A[imin]    // swap
```

Basic op

Picture:

$$\underbrace{A_1 \cdots A_{i-1}} \; A_i \; \underbrace{A_{i+1} \cdots A_n}_{\text{unsorted}}$$

- sorted
- $(i-1)$ smallest elements

Basic Operation: comparison of array elements

$$\#\text{comparisons} = (n-1) + (n-2) + \cdots + 2 + 1$$

$$= \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n = \Theta(n^2)$$

$$S = 1+2+3+ \cdots + (n-3)+(n-2)+(n-1)+n$$

$$S = n+(n-1)+(n-2)+ \cdots + 3 + 2 + 1$$

$$S+S = (n+1)+(n+1)+(n+1)+ \cdots + (n+1)+(n+1)+(n+1)$$

$$2S = n(n+1)$$

$$\therefore \boxed{S = \frac{n(n+1)}{2}}$$

$$n \leftarrow (n-1)$$

$$1+2+\cdots+(n-1) = \frac{(n-1)((n-\cancel{1})+\cancel{1})}{2}$$

$$= \frac{n(n-1)}{2}$$

<u>Part</u>: see Project description