

CSE 101 3-10-25

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- SETs : closes Sun. 11:59 PM

- Pa 8 : ext 2 days.

Heaps & Priority Queue

A Priority Queue is an ADT that maintains a finite set Q of records.

each record in \mathcal{S} has an attribute called its key

$$\begin{array}{c}
 \uparrow \\
 \text{Record}
 \end{array}
 x = (\underbrace{\cdot, \cdot, \cdot, \cdot}_{\text{satellite data}}, \text{key}) \in \mathcal{S}$$

$$\mathcal{S} = \{ \cdot, \cdot, x, \cdot, \cdot \}$$

\mathcal{S} is a state in the Priority queue. $\text{key}[x] = x.\text{key}$ defines the Priority of x in \mathcal{S} .

Two types: max P.Q.

min P.Q.

A max P.Q. has following

Operations

• $\text{Insert}(S, x)$

• $\text{Max}(S)$: pure access fn.

• $\text{ExtractMax}(S)$: returns & deletes
max key.

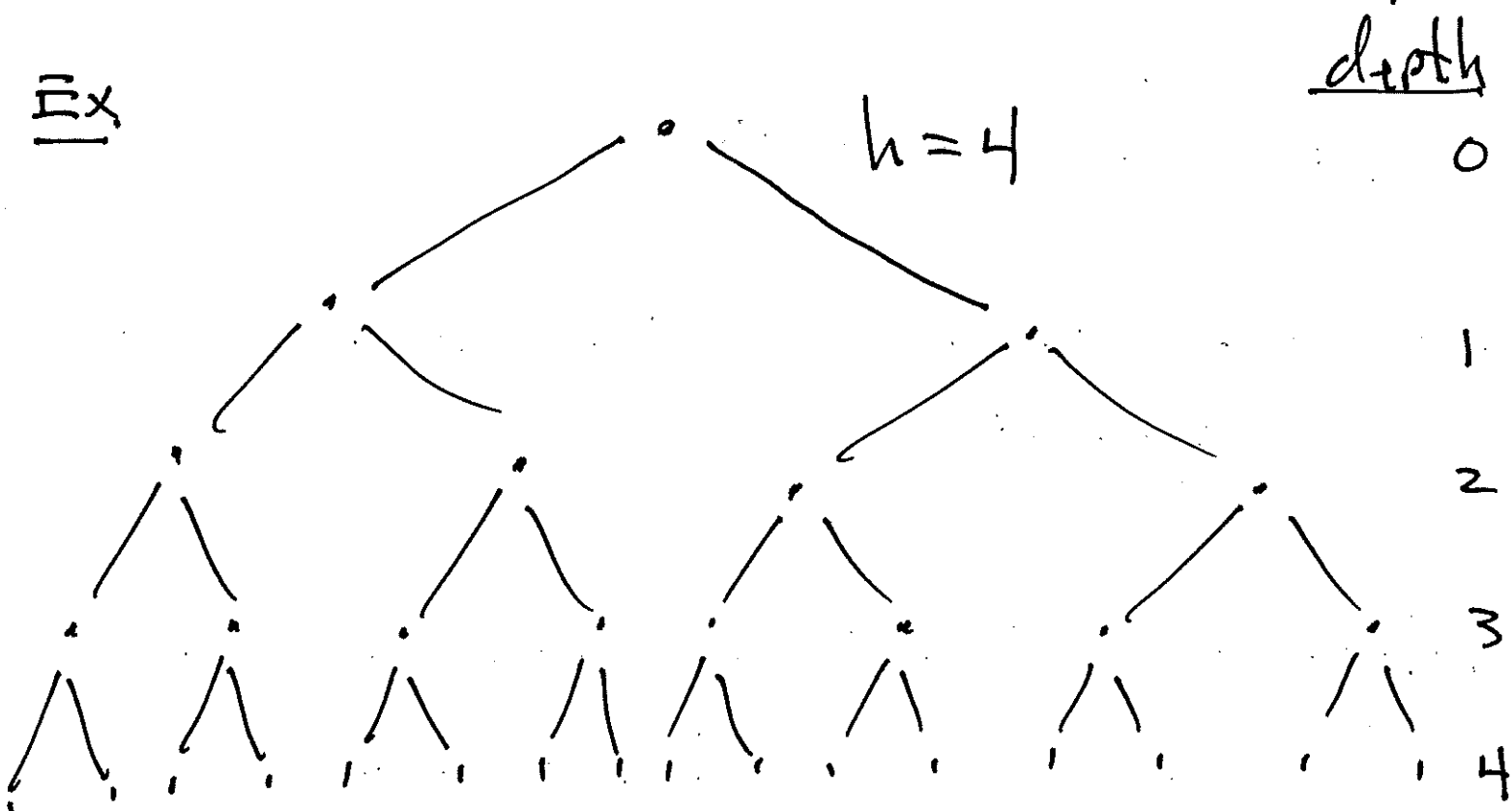
• $\text{IncreaseKey}(S, x, k)$: if $k > \text{key}[x]$,
change $\text{key}[x]$ to k otherwise
do nothing.

Heaps : chap 6

Defn A complete Binary Tree

(CBT) is a B.T. where all leaves have same depth, all internal nodes have ≥ 2 children

Ex



observe

$$(\# \text{ nodes at depth } d) = 2^d$$

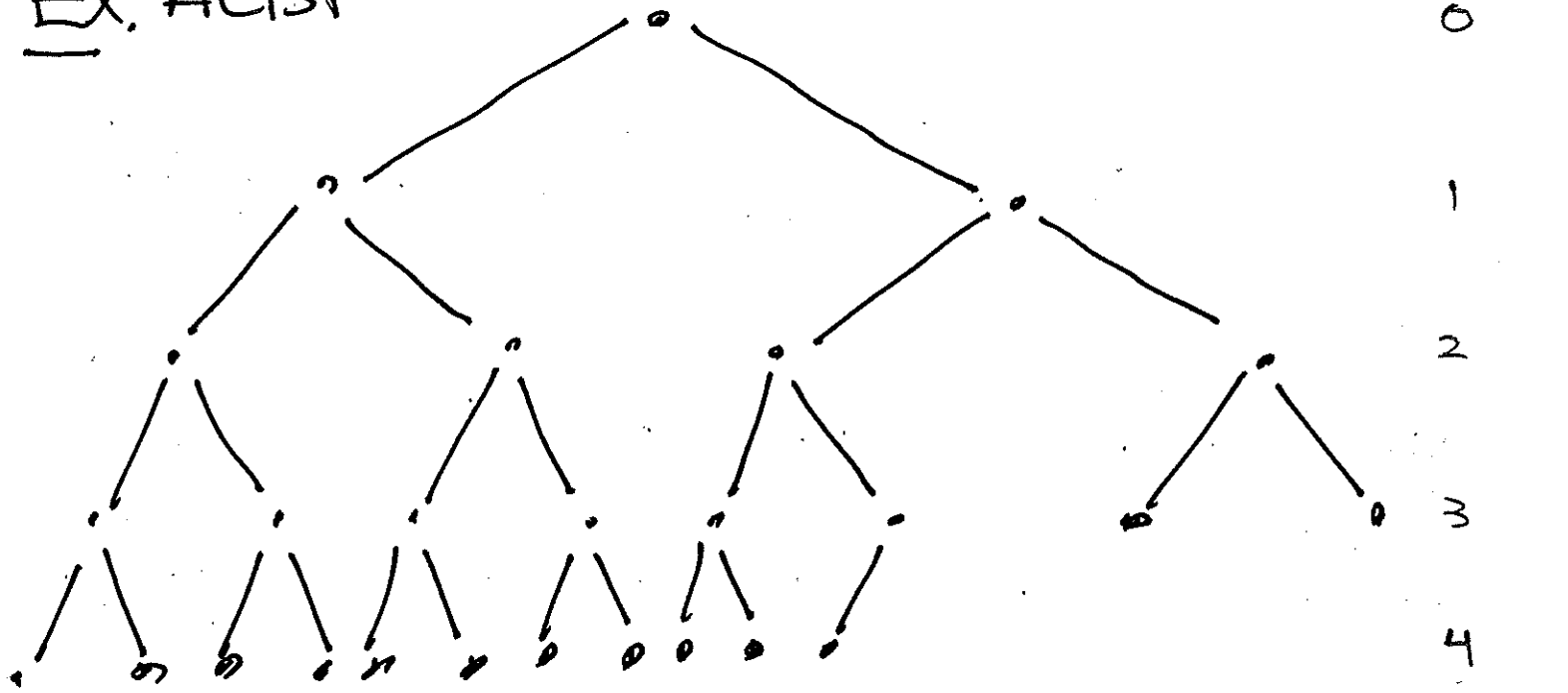
if $\#$ nodes in tree is n and height of tree is h , then

$$n = \sum_{d=0}^h 2^d = \frac{2^{h+1} - 1}{2 - 1} = 2^{h+1} - 1$$

An Almost Complete Binary Tree

(ACBT) is a $\overline{\text{B.T.}}$ that is filled at all levels, except possibly the deepest, which is filled from left to right.

Ex. ACBT



The number of nodes n in an ACBT of height h satisfies

$$2^h - 1 < n \leq 2^{h+1} - 1$$

$$\therefore 2^h \leq n < 2^{h+1}$$

$$\therefore h \leq \lfloor \lg(n) \rfloor < h+1$$

∴

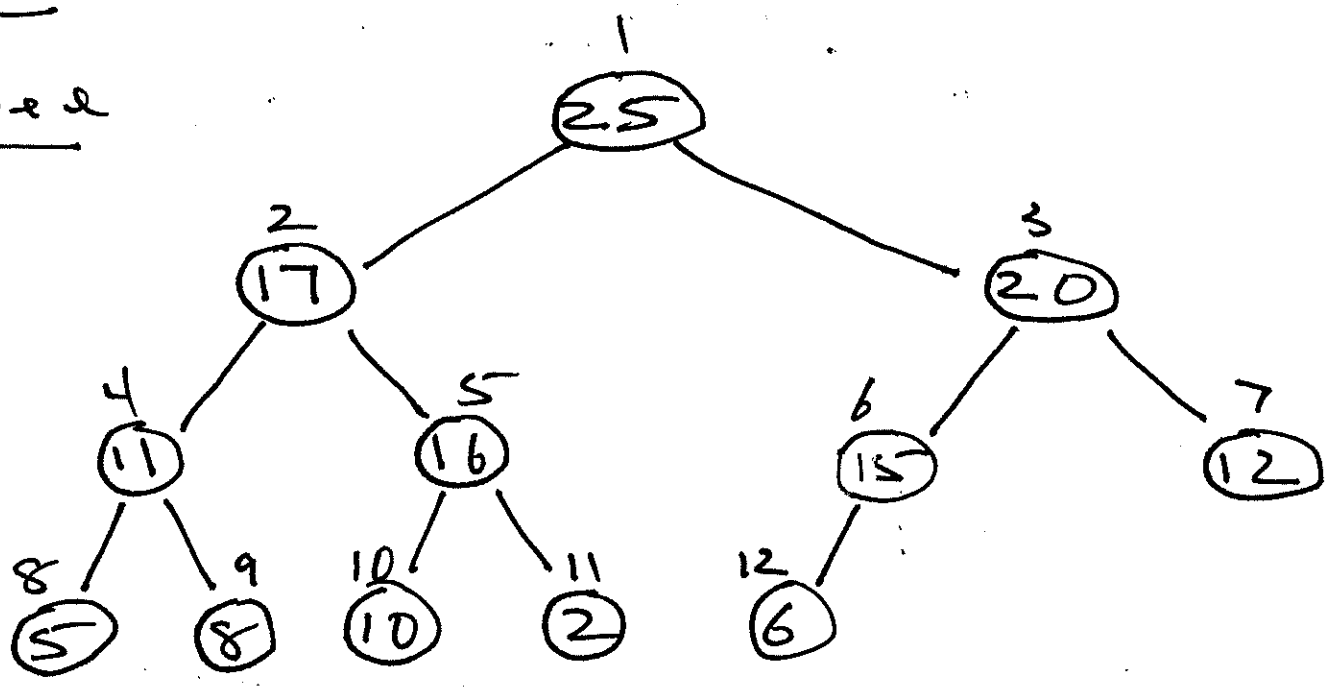
$$h = \lfloor \lg n \rfloor$$

Binary Heaps (max, min)

A heap is an array object that represents an ACBT.

Ex.

Tree



heapSize = 12 length = 16

Array

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	25	17	20	11	16	15	12	5	8	10	2	6

the array has 2 attributes

- length[A]
- heapSize[A]

• helper functions

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$$\text{Parent}(i) = \lfloor \frac{i}{2} \rfloor \quad (\text{for } i \geq 2)$$

$$\text{left}(i) = 2i$$

$$\text{right}(i) = 2i + 1$$

• heap Properties

$$\underline{\text{max}} \quad A[\text{Parent}(i)] \geq A[i] \quad \checkmark$$

$$\text{min} \quad A[\text{Parent}(i)] \leq A[i]$$