CSE 101 2-5-25

more generally:

- $\ln(n) = o(n^k)$ for $k > 0$, $k \in \mathbb{R}$.

Proof $\lim_{n\to\infty} \frac{\ln(n)}{n^k} = \lim_{n\to\infty} \frac{1/n}{k \cdot n^{k-1}}$

$= \frac{1}{k} \cdot \lim_{n\to\infty} \frac{1}{n^k} = 0$

∎

same for $\log_b(n)$ instead of $\ln(n)$.

Also

- $(\ln(n))^m = o(n^k)$.

$$\lim_{n\to\infty} \frac{(\ln(n))^m}{n^k} = \lim_{n\to\infty} \frac{m(\ln(n))^{m-1}\cdot\frac{1}{n}}{k n^{k-1}}$$

$$= \frac{m}{k}\cdot\lim_{n\to\infty} \frac{(\ln(n))^{m-1}}{n^k}$$

$\lceil m \rceil$ applications of l'HoP.

$$= \text{const}\cdot\lim_{n\to\infty} \frac{1}{n^k}$$

Ex. $f(n) = n^3$, $g(n) = n^4$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^3}{n^4}$$

$$= \lim_{n\to\infty} \frac{1}{n} = 0$$

$\therefore \quad n^3 = o(n^4)$

Ex. $f(n) = n^k$ $(k \geq 0)$, $g(n) = e^n$

Same for b^n

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^k}{e^n}$$

$$= k \lim_{n\to\infty} \frac{n^{k-1}}{e^n}$$

$\vdots$ $\lceil k \rceil$ app. of l'Hop.

$= 0$

Also

<u>Ex.</u> $f(n) = a^n$, $g(n) = b^n$, $1 < a < b$

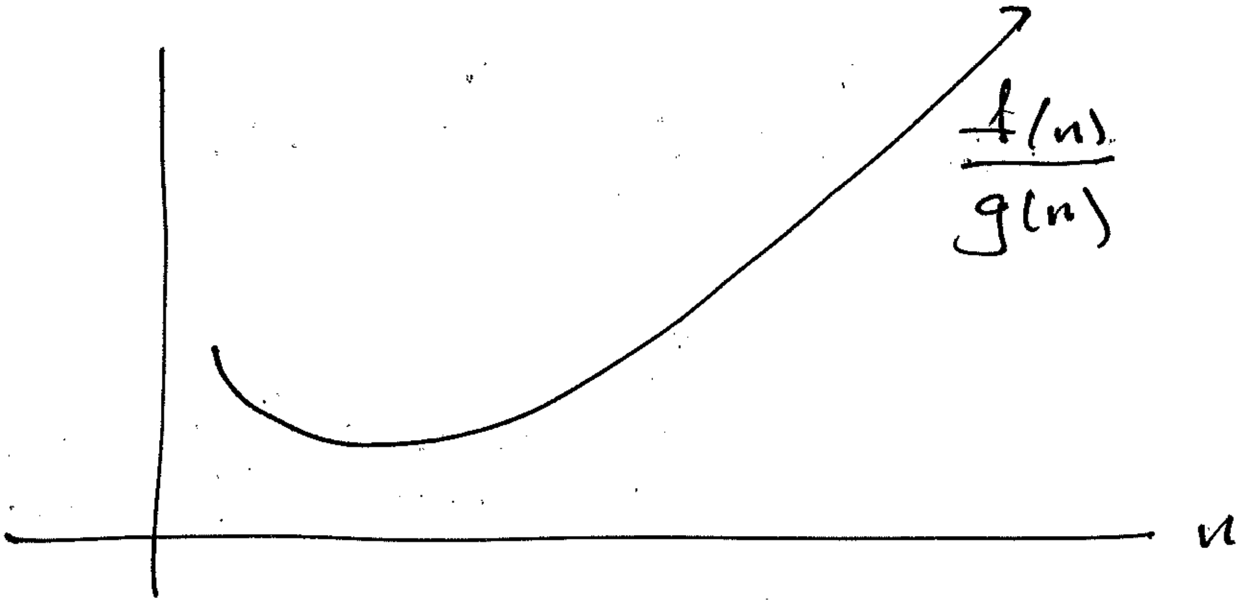
$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{a^n}{b^n}$$

$$= \lim_{n\to\infty} \left(\frac{a}{b}\right)^n = 0$$

$\therefore \quad a^n = o(b^n)$.

<u>Defn</u>

$f(n) = \omega(g(n))$ iff $\displaystyle\lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right) = \infty$.



Recall: Limit of reciprocal is reciprocal of limit. Thus

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \quad \text{iff} \quad \lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty$$

Thus

$$f(n) = o(g(n)) \text{ iff } g(n) = \omega(f(n)).$$

Analogy:

$f(n) = O(g(n)) \quad \sim \quad x \le y$

$f = \Omega(g) \quad \sim \quad x \ge y$

$f = \Theta(g) \quad \sim \quad x = y$

$f = o(g) \quad \sim \quad x < y$

$f = \omega(g) \quad \sim \quad x > y$

note

$f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

$\frac{f(n)}{g(n)} \le B$ iff $\frac{g(n)}{f(n)} \ge \frac{1}{B}$

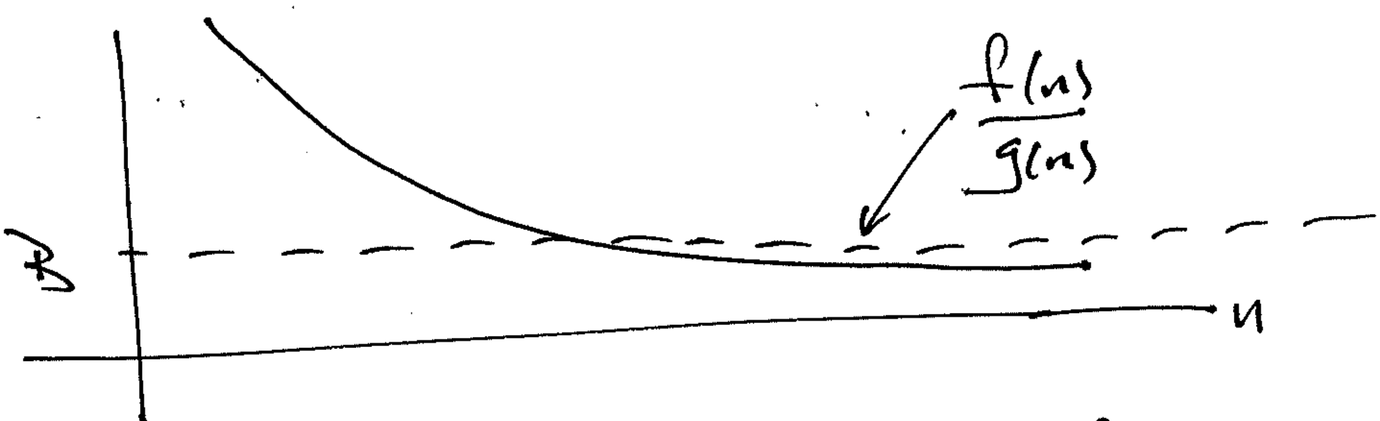
(for $n \ge n_0$) (for $n \ge n_0$)

note:

$$f(n) = o(g(n)) \overset{\not\Leftarrow}{\Rightarrow} f(n) = O(g(n))$$

Pf:

$$f = o(g) \Rightarrow \lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = 0$$



Thus there is a const. $B > 0$

s.t. $\frac{f(n)}{g(n)} \le B$ (for $n \ge n_0$, some n_0)

$\therefore f(n) = O(g(n))$ ∎

<u>also</u>: transitivity

$f(n) = O(g(n))$ and $g(n) = O(h(n))$
implies $f(n) = O(h(n))$.

<u>Exercise</u>.

See p. 8 of handout.

#3 for any $c > 0$

$$c \cdot f(n) = O(f(n))$$
$$c \cdot f(n) = \Omega(f(n))$$
$$c \cdot f(n) = \Theta(f(n))$$
$$f(n) = o(g(n)) \Rightarrow c \cdot f(n) = o(g(n))$$
$$\omega \Rightarrow \omega$$

#1] $\alpha, \beta \in \mathbb{R}$

$$n^{\alpha} = \begin{cases} o(n^{\beta}) & \text{if } \alpha < \beta \\ \Theta(n^{\beta}) & \alpha = \beta \\ \omega(n^{\beta}) & \alpha > \beta \end{cases}$$

$$\frac{n^{\alpha}}{n^{\beta}} = n^{\alpha-\beta} \longrightarrow \begin{cases} 0 & \alpha < \beta \\ 1 & \alpha = \beta \\ \infty & \alpha > \beta \end{cases}$$

why?

#5] let $L = \lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right)$, if it exists.

a.) $0 \le L < \infty \Rightarrow f(n) = O(g(n))$

b.) $0 < L \le \infty \Rightarrow f(n) = \Omega(g(n))$

c.) $0 < L < \infty \Rightarrow f(n) = \Theta(g(n))$

d.) $L = 0 \iff f(n) = o(g(n))$

e.) $L = \infty \iff f(n) = \omega(g(n))$

#4) if $a > 1$, $b > 1$, then

$$\log_a(n) = \Theta(\log_b(n))$$.

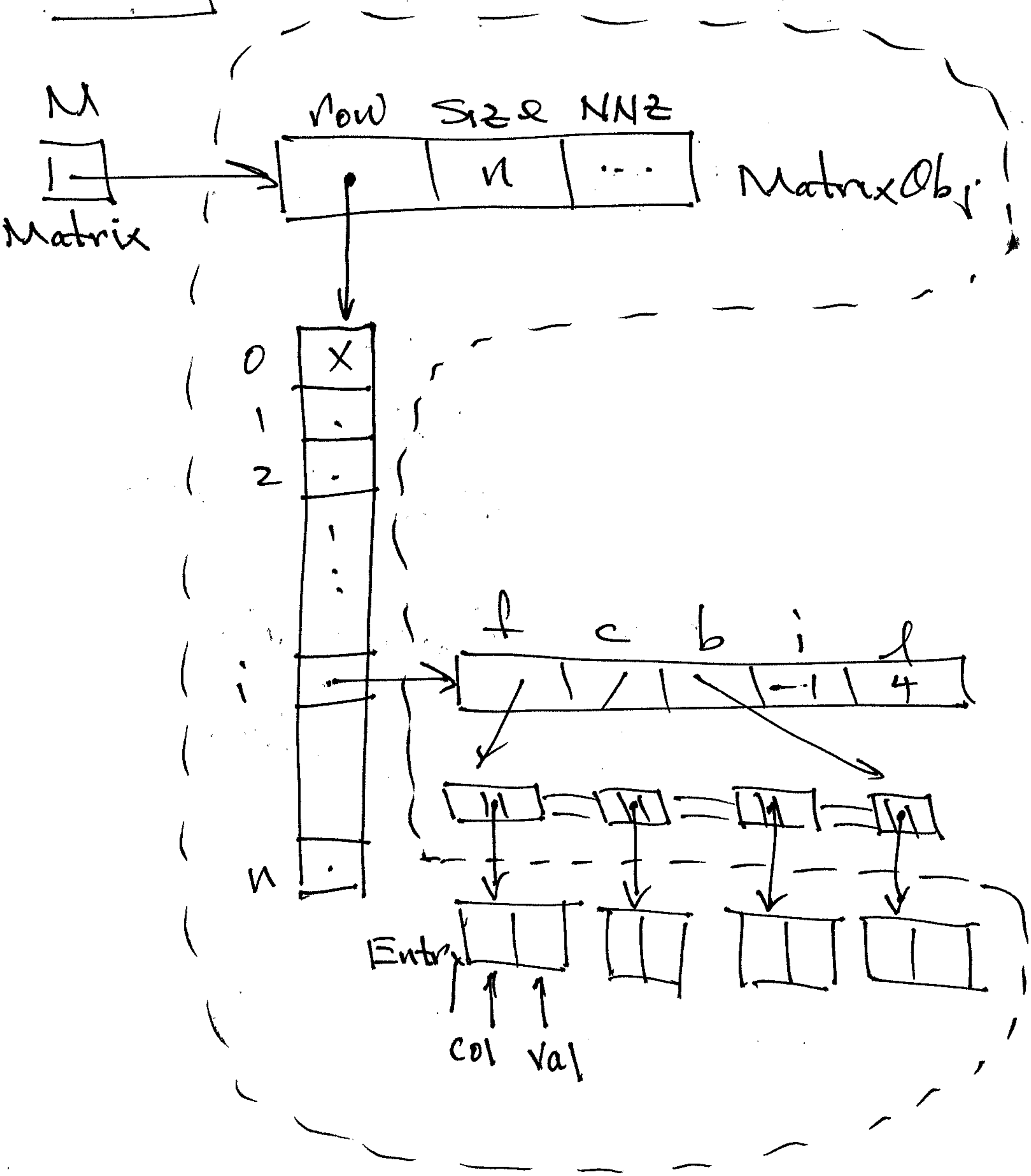
why? since

$$\log_a(n) = \frac{\ln(n)}{\ln(b)}$$

$$\therefore \underbrace{\ln(b)}_{\text{const.}} \cdot \log_a(n) = \ln(n)$$

more to come ....

Pa4


M
1
Matrix
row
size
NNZ
n
...
MatrixObj
0
X
1
2
i
n
f
c
b
i
l
-1
4
Entry
col
val