CSE 101 2-3-25

- mid 1 solutions posted
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<u>Handout</u>: Algorithm Runtime

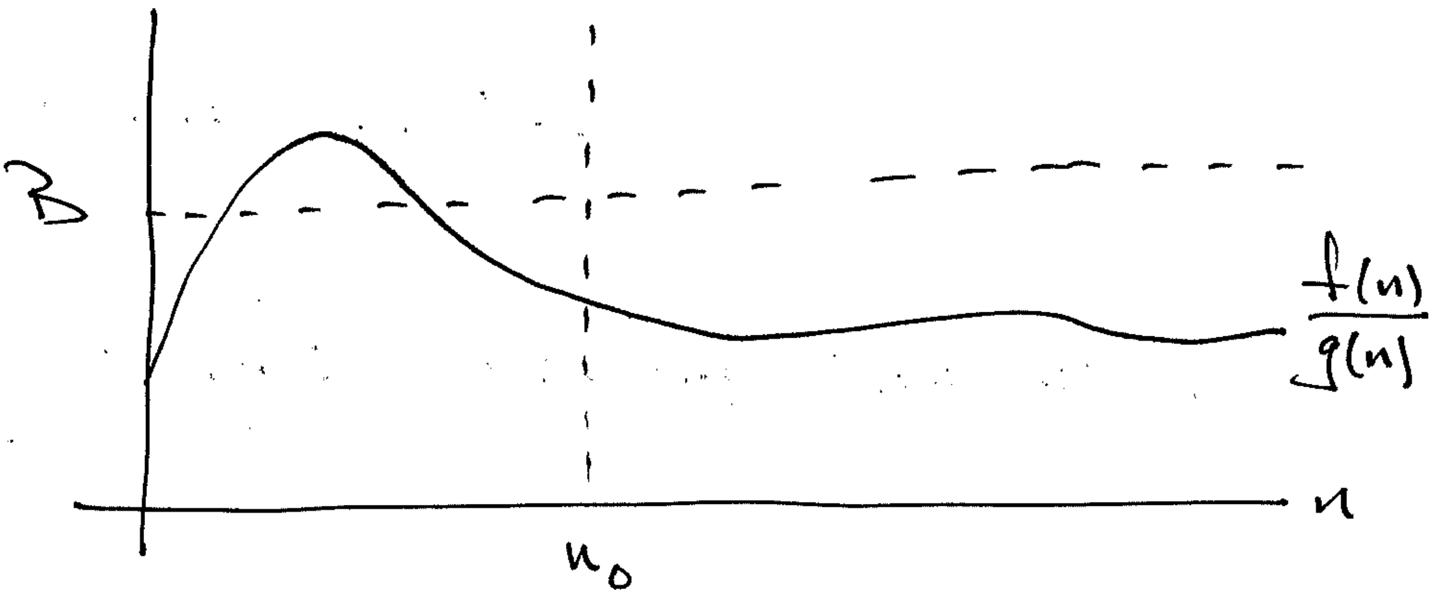
<u>Defn</u>

We say $f(n) = O(g(n))$ iff there exist $B > 0$, $n_0 > 0$ such that

$$\frac{f(n)}{g(n)} \le B$$

for all $n \ge n_0$.

- $g(n)$ is an <u>asymptotic upper bound</u> for $f(n)$



<u>Ex</u> $f(n) = 2n+5$, $g(n) = n$. so

$$\frac{f(n)}{g(n)} = 2 + \frac{5}{n} \le \underset{B}{3}$$

for all $n \ge \underset{n_0}{5}$

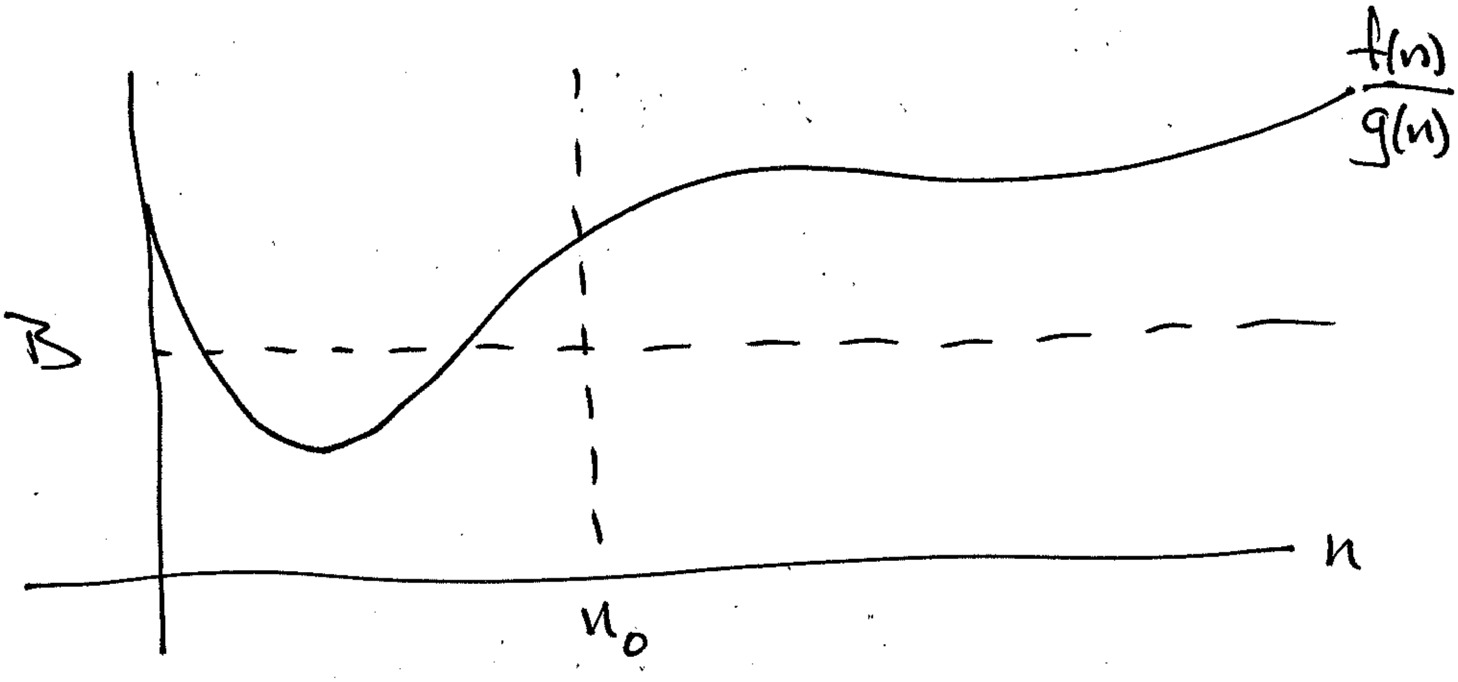
In fact: if $f(n)$ is a d^{th} degree Polynomial, then $f(n) = O(n^d)$.

<u>Defn</u>

We say $f(n) = \Omega(g(n))$ iff there exist $B > 0$, $n_0 > 0$ s.t.

$$B \leq \frac{f(n)}{g(n)}$$

for all $n \geq n_0$



Ex. $f(n) = 6n^3 + 4n$, $g(n) = 2n^2$.

$$\frac{f(n)}{g(n)} = 3n + \frac{2}{n} \geq \underset{\substack{\uparrow \\ B}}{7}$$

for all $n \geq \underset{\substack{\uparrow \\ n_0}}{2}$

Fact: if $f(n)$ is a d^{th} deg. Poly and $g(n)$ is a e^{th} deg. Poly, then

$$d \le e \Rightarrow f(n) = O(g(n))$$

and

$$e \le d \Rightarrow f(n) = \Omega(g(n))$$

Defn

We say $f(n) = \Theta(g(n))$ iff there exist $B_1 > 0$, $B_2 > 0$, $n_0 > 0$ s.t.

$$B_1 \le \frac{f(n)}{g(n)} \le B_2$$

for all $n \ge n_0$.

<u>Ex.</u> $f(n) = 5n^2 + 11n - 24$, $g(n) = n^2$.

Then

$$4 \le \frac{5n^2 + 11n - 24}{n^2} \le 6$$

for all $n \ge 8$. (exercise! Prove this)

<u>Fact:</u> if $f(n), g(n)$ are Poly., then

$\deg(f) \le \deg(g) \Rightarrow f = O(g)$

" $\ge$ " $\Rightarrow f = \Omega(g)$

" $=$ " $\Rightarrow f = \Theta(g)$.

Analogy

$x \le y \sim f(n) = O(g(n))$

$x \ge y \sim f(n) = \Omega(g(n))$

$x = y \sim f(n) = \Theta(g(n))$

$x < y$?

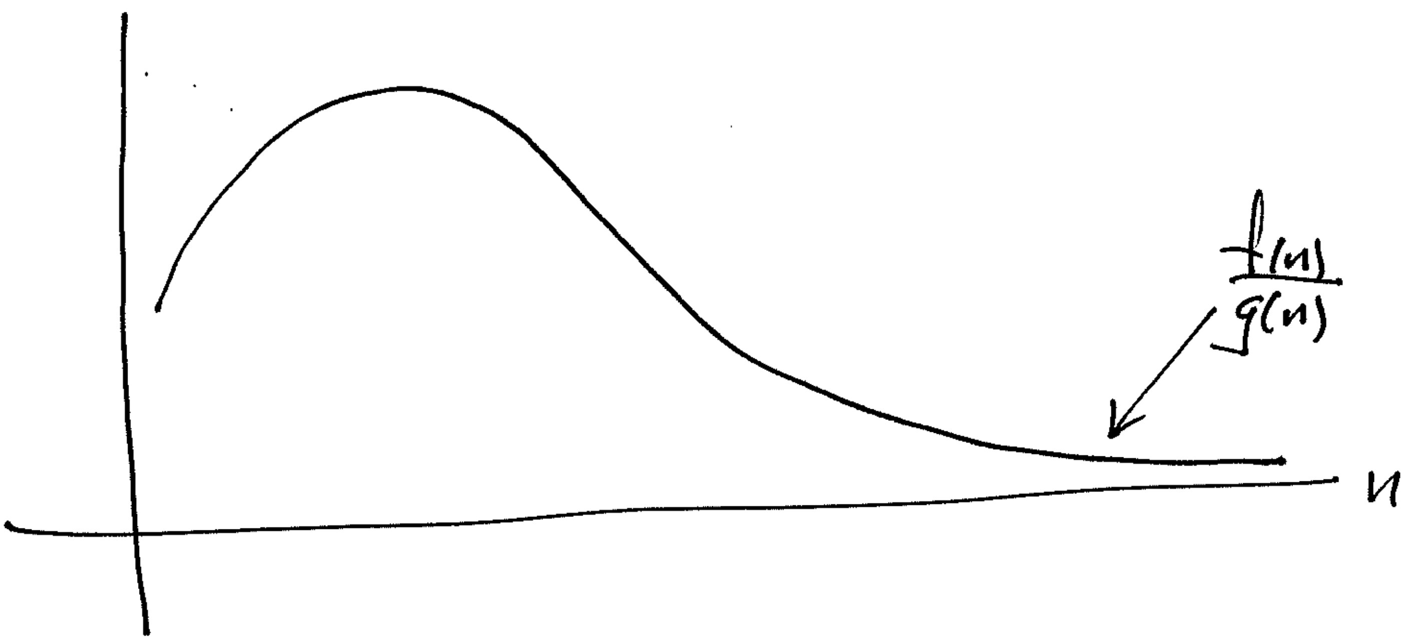
$x > y$?

Defn

$f(n) = o(g(n))$ iff

$$\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = 0$$

we say $g(n)$ is a strict asymptotic upper bound for $f(n)$



note: $f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$.

<u>Ex.</u> $f(n) = \ln(n)$, $g(n) = n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\ln(n)}{n}$$

$$= \lim_{n\to\infty} \frac{1/n}{1}$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right) = 0 \checkmark$$

$$\therefore \ln(n) = o(n)$$