CSE 101 1-15-25

Defn

A Graph $G = (V, E)$ where

$V = \{\text{vertex set}\}$

$E = \{\text{some unordered pairs in } V\}$

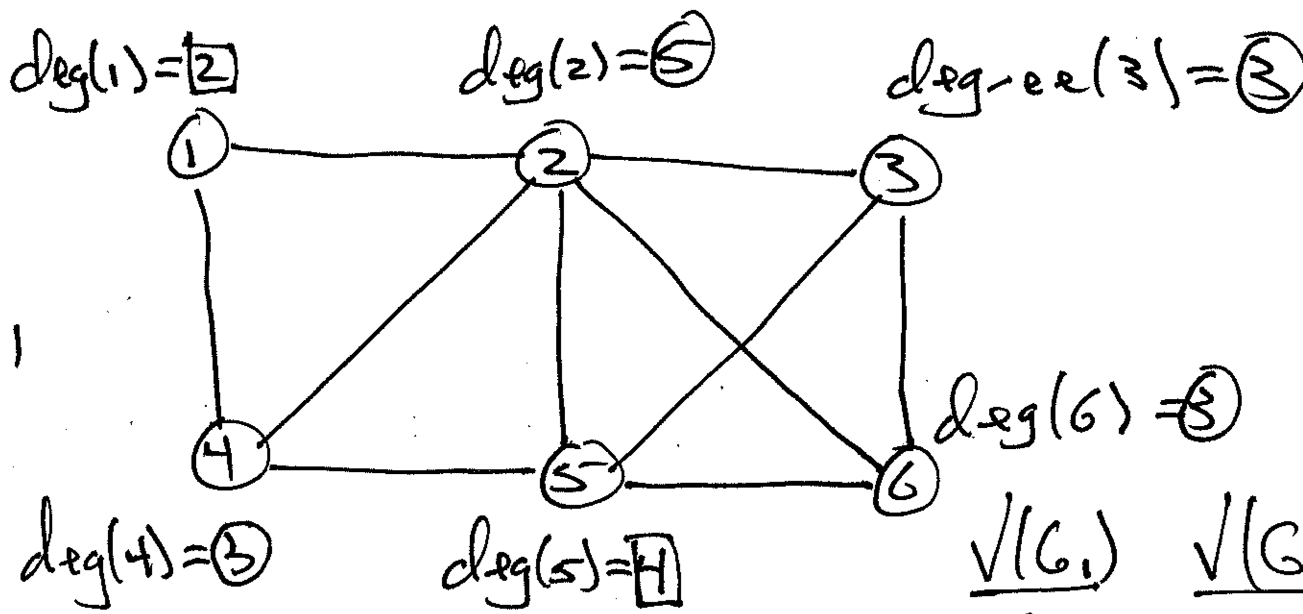
$\subseteq V^{(2)}$

Ex. $V = \{1, 2, 3, 4, 5, 6\}$

$E = \{12, 14, 23, 24, 25, 26, 35, 36, 45, 56\}$

$\{x, y\} = xy$

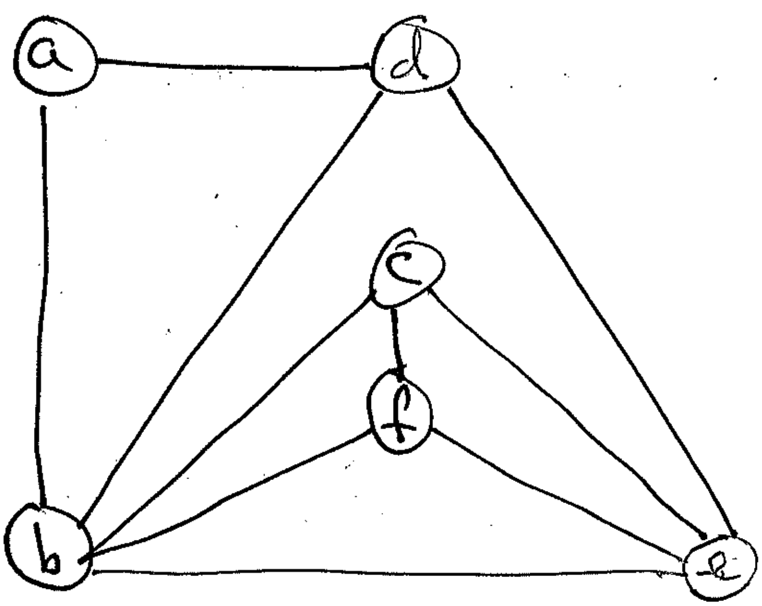
1, 2 are adjacent
12, 23 " "



G_1

$V(G_1)$	$V(G_2)$
1 $\longrightarrow$	a
2 $\longrightarrow$	b
3 $\longrightarrow$	c
4 $\longrightarrow$	d
5 $\longrightarrow$	e
6 $\longrightarrow$	f

Ex.



G_2

$V = \{a, b, c, d, e, f\}$

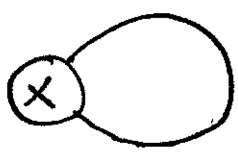
$E = \{ab, ad, bc, bd, bf, be, cf, ce, de, fe\}$

An isomorphism is a mapping (bijective) ϕ

$V(G_1) \xrightarrow{\phi} V(G_2)$ such that

$$\{x, y\} \in E(G_1) \iff \{\phi(x), \phi(y)\} \in E(G_2)$$

note:

self-loop  not allowed

Parallel edges  not allowed observe: <u>Handshake Lemma</u>

$$\sum_{x \in V(G)} \deg(x) = 2|E(G)|$$

<u>Also</u>: the # of odd degree vertices in a graph must be even.

<u>Defns</u>.

- an x-y walk is a seq. of vertices

$$x = v_0, v_1, v_2, \ldots, v_k = y$$

such that $\{v_{i-1}, v_i\} \in E$. The length is k, the # edges.

Ex. A 1-6 walk: 1, 2, 4, 5, 2, 3, 6
length = 6

Defn

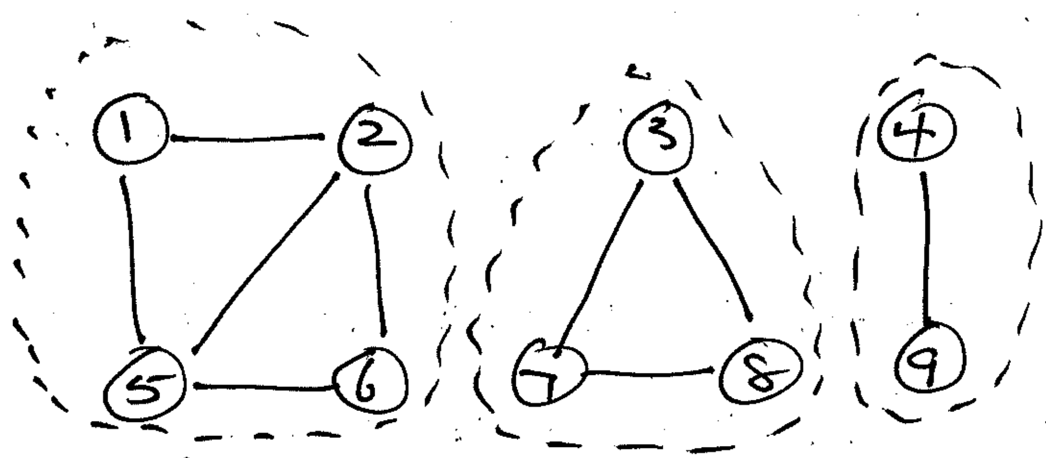
- if $x = y$, we say walk is closed. if length = 0, we call the walk trivial.
- a walk in which no edge is traversed multiple times is called a trail.
- a trail in which no vertex is visited more than once (except if closed) is called a path.

- a non-trivial closed path is called a <u>cycle</u>.

<u>Defn</u>

$G=(V,E)$ is called <u>connected</u> iff for all $x, y \in V$, G contains an x-y path. Otherwise G is <u>Disconnected</u>

<u>Ex.</u>



$E = \{12, 15, 25, 26, 56, 37, 38, 78, 49\}$

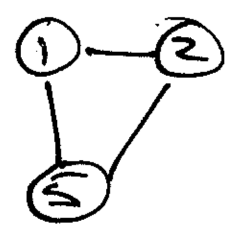
Defn

A subgraph H of G is a graph $H=(V',E')$ such that

$$V' \subseteq V(G)$$
$$E' \subseteq E(G)$$

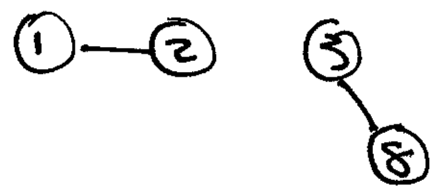
Ex.

- $(\{1,2,5\},\{12,25,15\})$



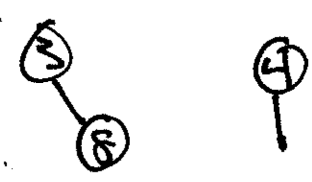
connected

- $(\{1,2,3,8\},\{12,38\})$



dis-conn.

- $(\{3,8,4\},\{38,49\})$



not a graph, so not a subgraph.

Defn

A connected component in a Graph G is subgraph H of G such that

(i) H is connected

(ii) H is maximal w.r.t. Property (i)