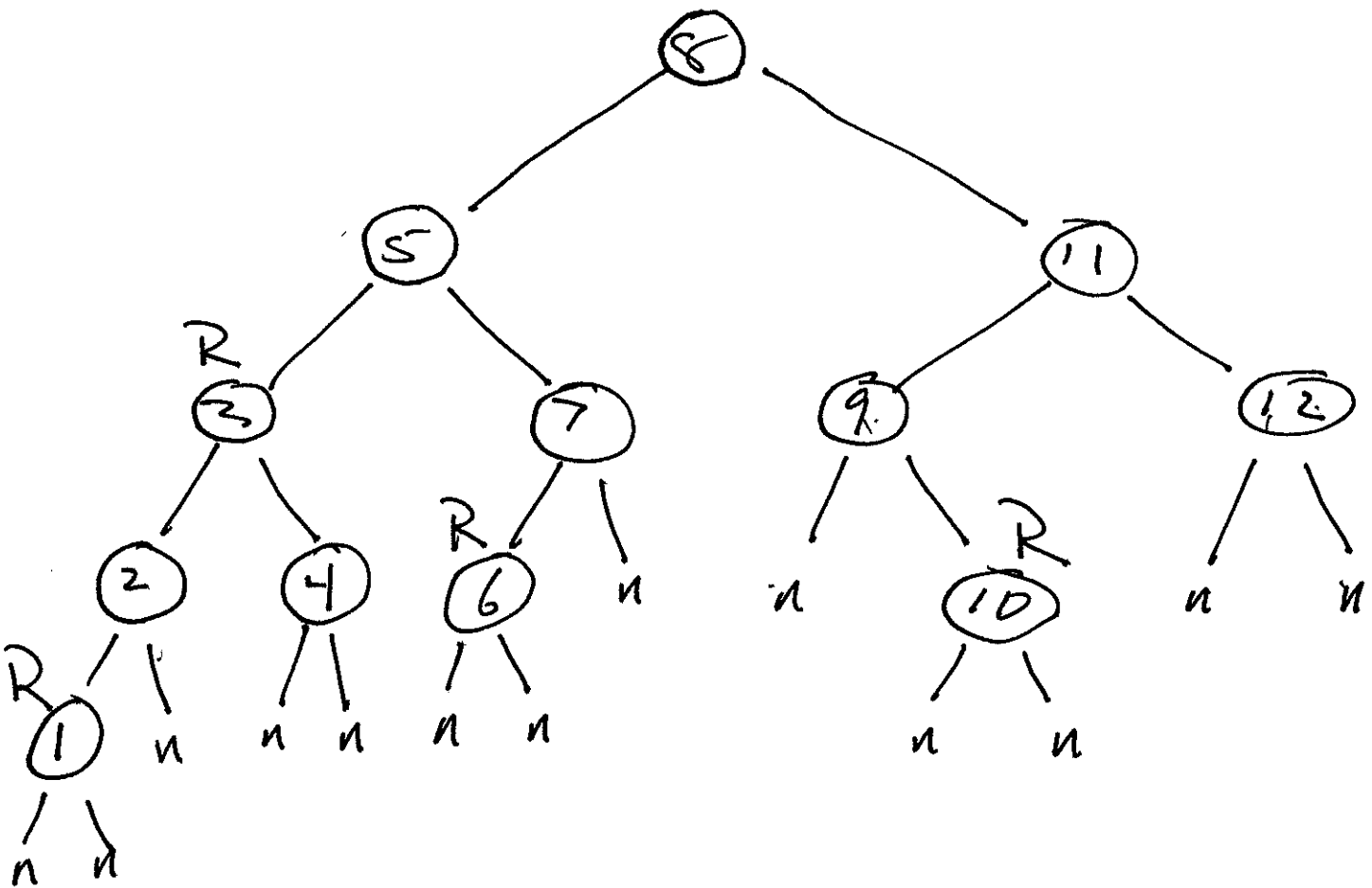


Q92 101 3-7-24

1

Part: ext. 1 last day to Saturday 10 PM.

RTS: example



<u>nodes</u>	<u>black height</u>
8	3
5, 11, 3	2
1, 2, 4, 6, 7, 9, 10, 12	1
all nils	0

note: $bh(x) = 0$ iff $height(x) = 0$
 iff x is a leaf/nil.

Theorem

A RBT with n internal (i.e. non-nil) nodes and height h satisfies

$$h \leq 2 \lfloor \lg(n+1) \rfloor$$

Remark:

• in a RBT

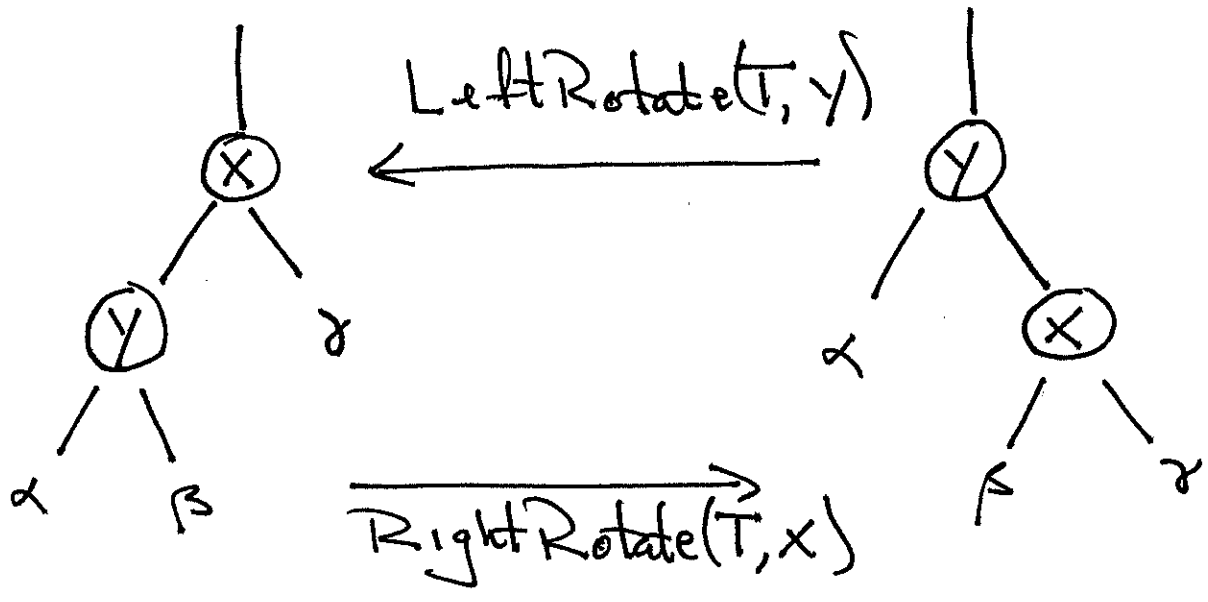
$$\lfloor \lg(n) \rfloor \leq \text{height}(T) \leq 2 \lfloor \lg(n+1) \rfloor$$

$$\circ \circ \text{height}(T) = \Theta(\lg n)$$

• • queries $\frac{1}{2}$, inserts $\frac{1}{2}$, deletes
run in time $\Theta(\lg n)$.

B.2 Rotations

Picture



note: $key(\alpha) \leq key(Y) \leq key(\beta) \leq key(X) \leq key(\delta)$

Both Preserve BST Property.

Summarize RightRotate(T, x)

$$\uparrow \left\{ \begin{array}{l} \bullet x.\text{left} = y.\text{right} \\ \bullet y.\text{right}.\text{Parent} = x \end{array} \right.$$

$$\text{Parent} \left\{ \begin{array}{l} \bullet y.\text{Parent} = x.\text{Parent} \\ \bullet x.\text{Parent} \cdot \left\{ \begin{array}{l} \text{right} \\ \text{or} \\ \text{left} \end{array} \right\} = y \end{array} \right.$$

$$x, y \left\{ \begin{array}{l} \bullet y.\text{right} = x \\ \bullet x.\text{Parent} = y \end{array} \right.$$

Runtime: $\Theta(1)$

13.3 Insertion

- do BST insert of new node z
- color z Red.

◦ fix RBT Prop. 4

- color root black

How?

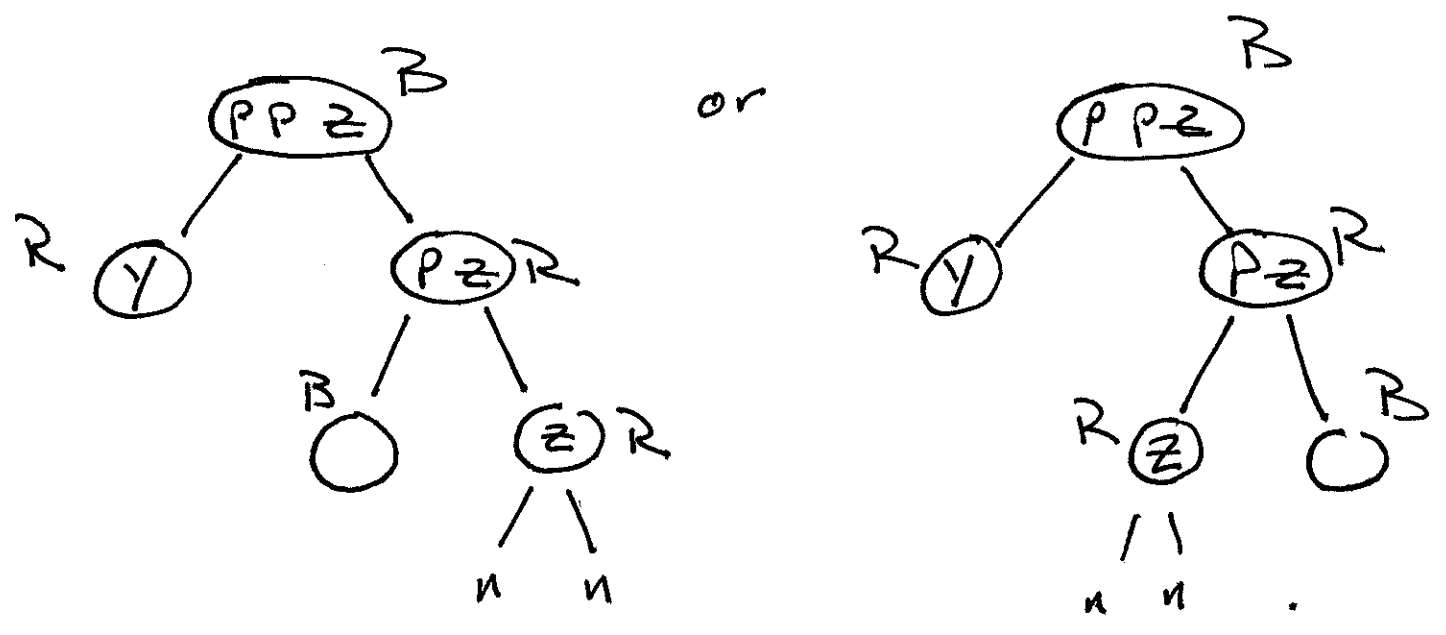
$RBInsertFixup(T, z)$

cases 4, 5, 6 of Fixup!

$z.Parent == z.Parent.Parent.Right$

let $y = z.Parent.Parent.Left$

case 4! y is Red



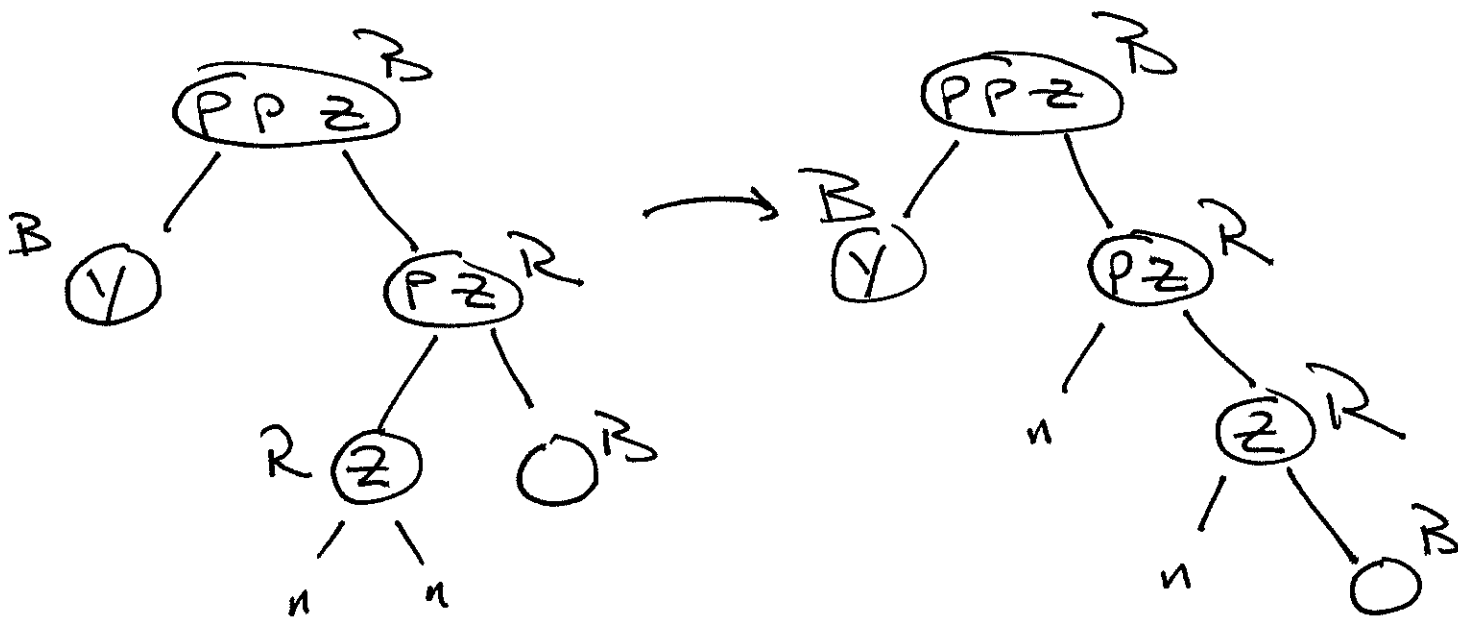
fix color relations between $y, Pz, PPz,$
let z climb up to PPz

Case 5: y is Black and

$z.parent.left == z$

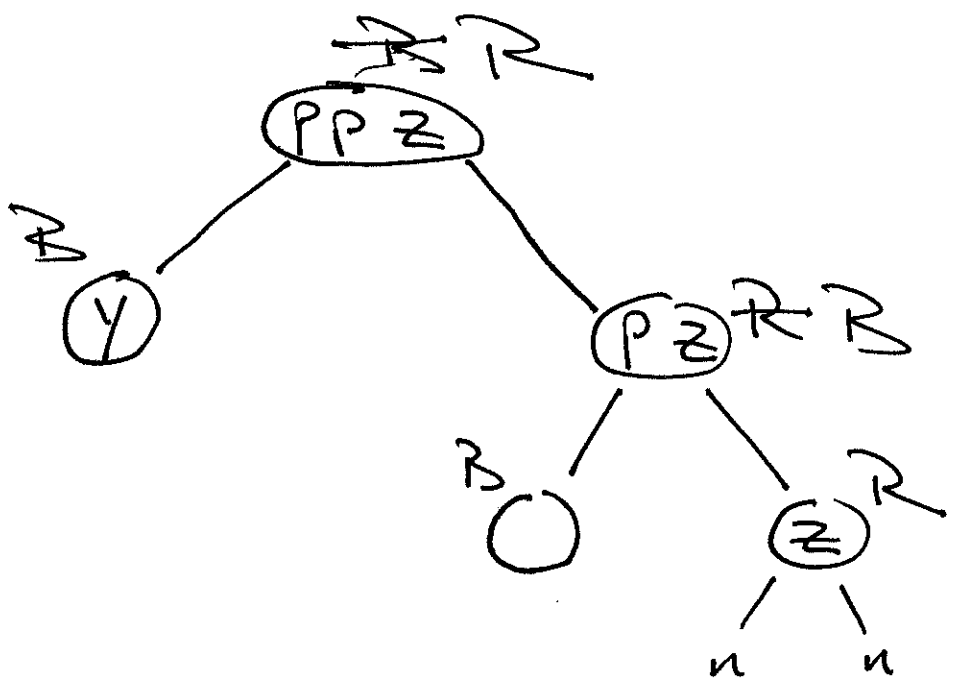
let z climb up to $z.parent$, then

right rotate about z

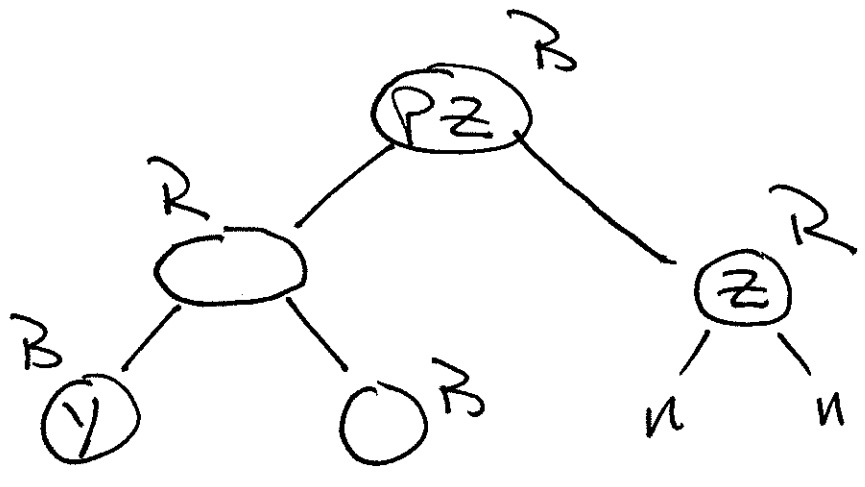


Case 6: y is Black

z . Parent . right == z



color PZ Black, color PPZ Red,
then left rotate about PPZ

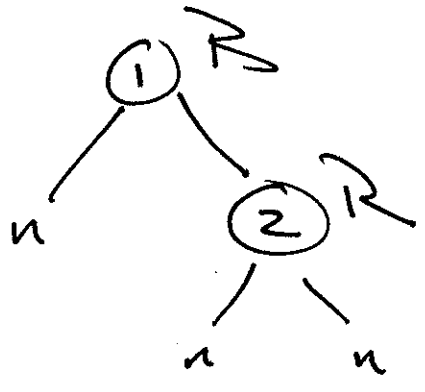


EX insert: 1, 2, 3, 4, 5

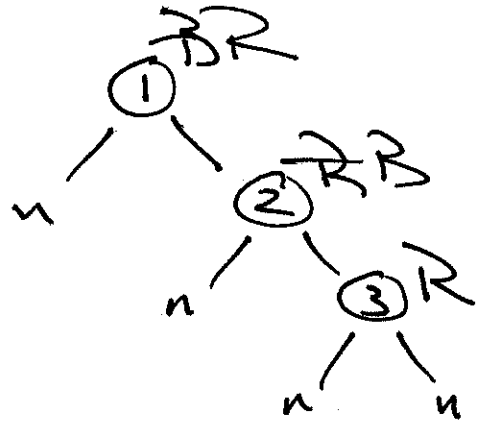
insert 1



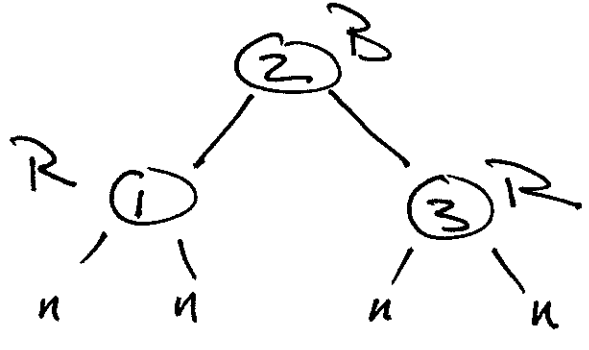
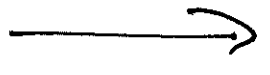
insert 2



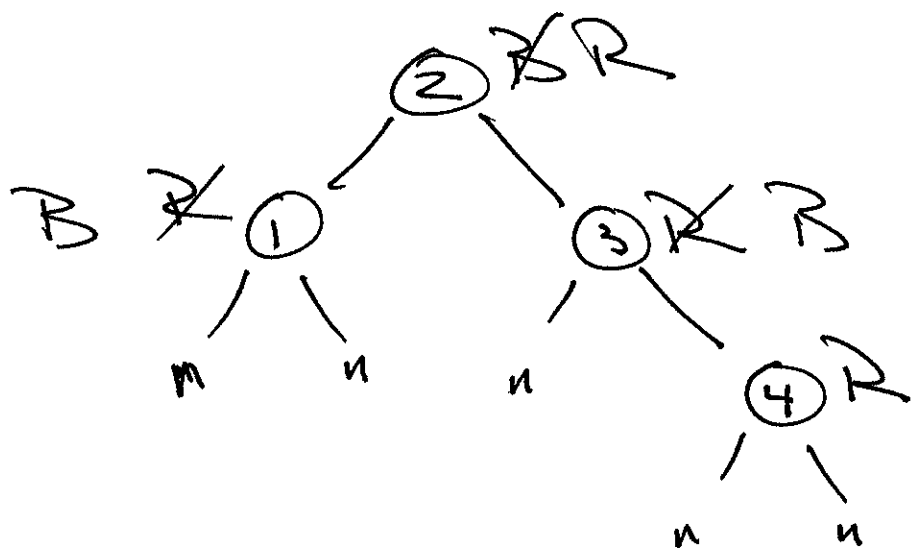
insert 3



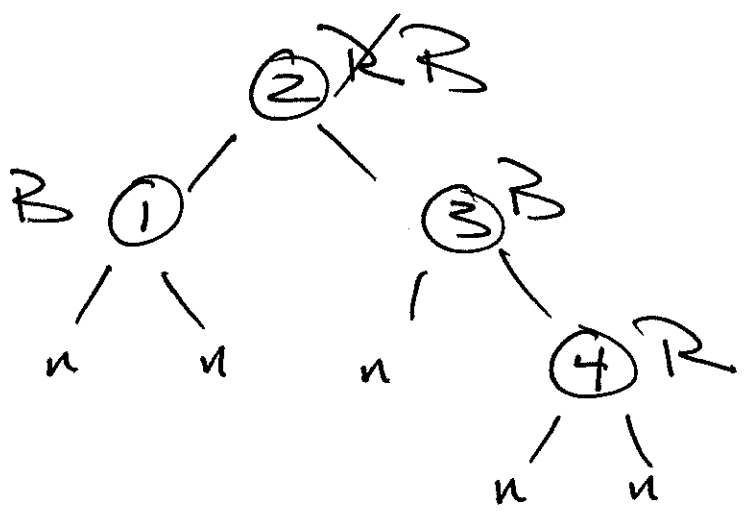
Case 6



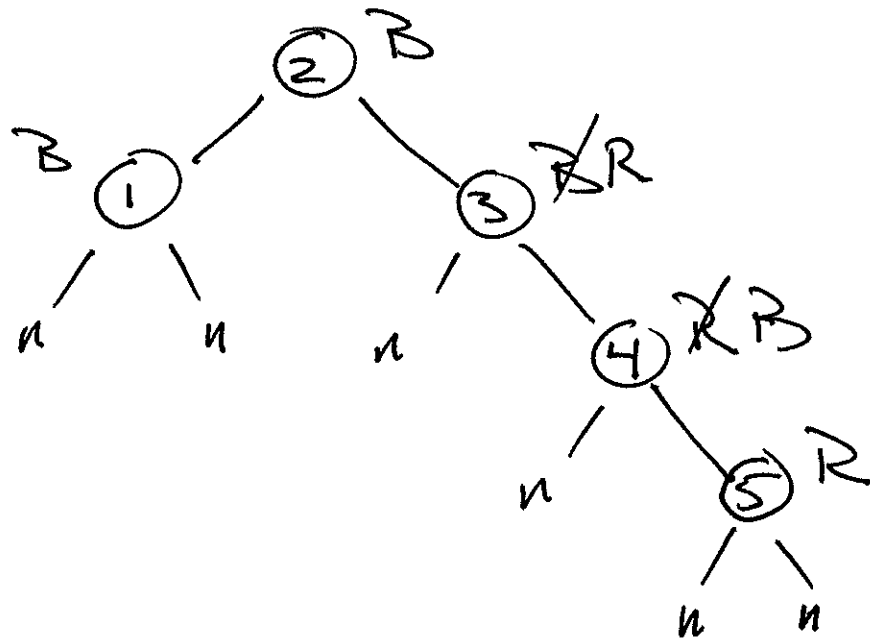
insert 4



Case 4



insert 5



Case 6

