cse 101 2-6-24

- PA3 due Wed. last extension

now compare:

Ex. for i=1 to n
    for i=1 to n
        for i=1 to n
            op.

$\Big]\ n^3$

Ex.

for i=1 to n
    for i=1 to n
        for i=1 to n
            op
            op

$\Big]\ 2n^3$

why consider $n^3$ and $2n^3$ to be the same?

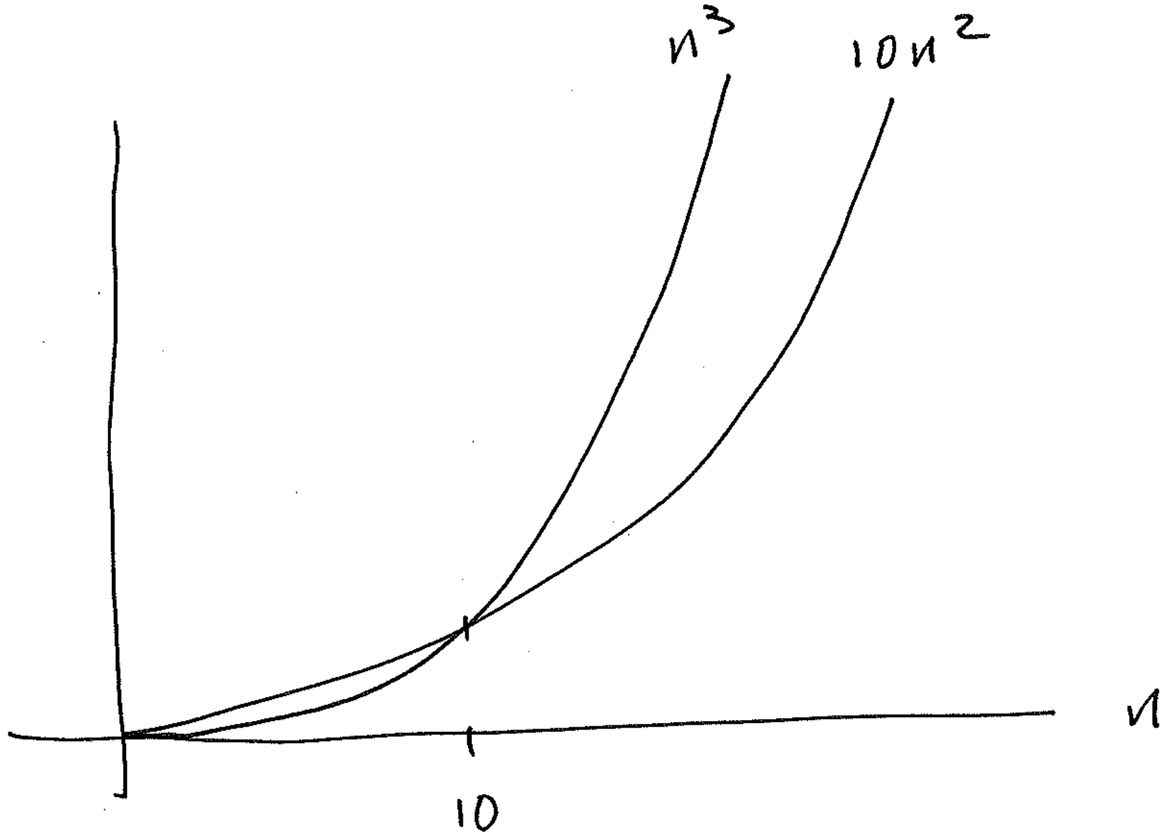
Because they can be equalized by running $2n^3$ on a faster computer.

Ex. compare:

A: $n^3$

to B: $10n^2$

crossover point: $n^3 = 10n^2 \Rightarrow n = 10$




## Informal Procedure

(1) choose a basic op. (inside innermost loop) and (2) count # times it's executed as a function of input size: $n$, then

(3) simplify function in (2) by dropping lower order terms and replace lead coefficient by 1.

# Asymptotic Growth of Functions

Let $f(n), g(n)$ be positive functions.

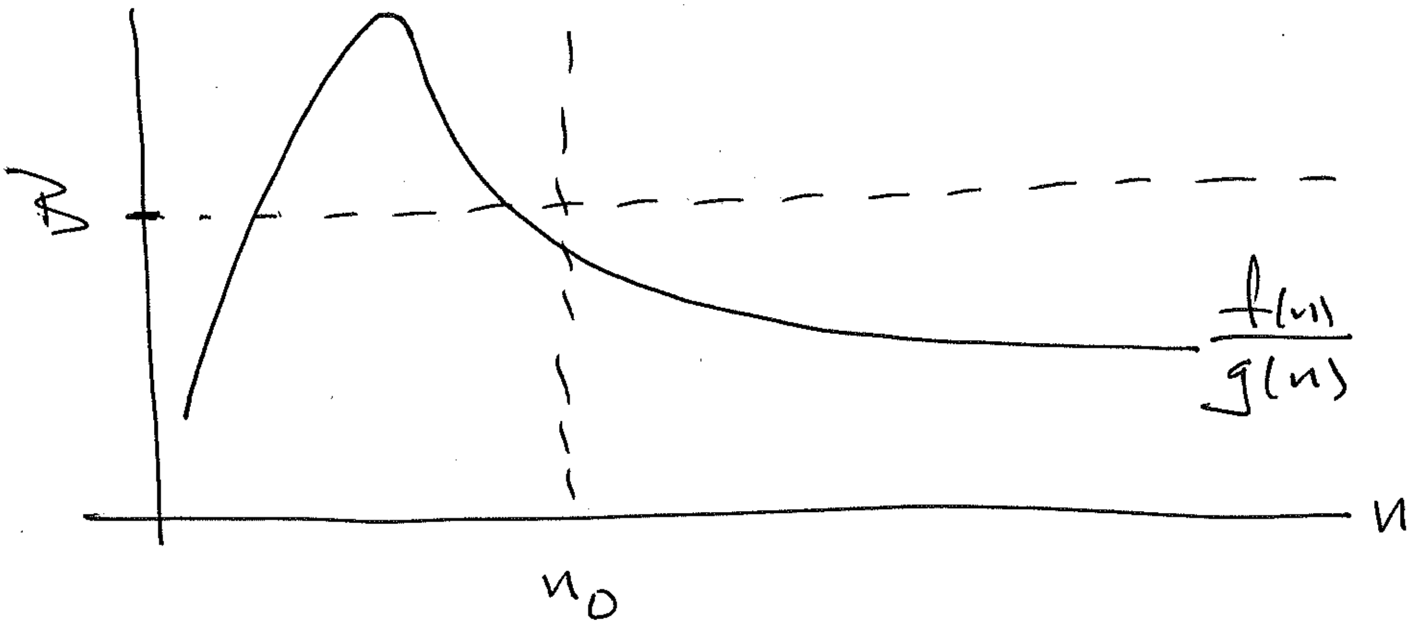
Defn

$f(n) = O(g(n))$ iff there exists $B > 0$, and $n_0 > 0$ such that

$$\frac{f(n)}{g(n)} \le B$$

for all $n \ge n_0$. We say $g(n)$ is an asymptotic upper bound for $f(n)$

meaning: $f(n)$ grows no faster than $g(n)$




<u>EX</u> $f(n) = 2n+5$, $g(n) = n$. <u>claim</u> $2n+5 = O(n)$

why?

$$\frac{f(n)}{g(n)} = 2 + \frac{5}{n} \le \underset{B}{\underset{\uparrow}{3}} \text{ for all } n \ge \underset{n_0}{\underset{\uparrow}{5}}$$

<u>Generalize</u> $an+b = O(n)$ for any $a>0$, and any $b$.

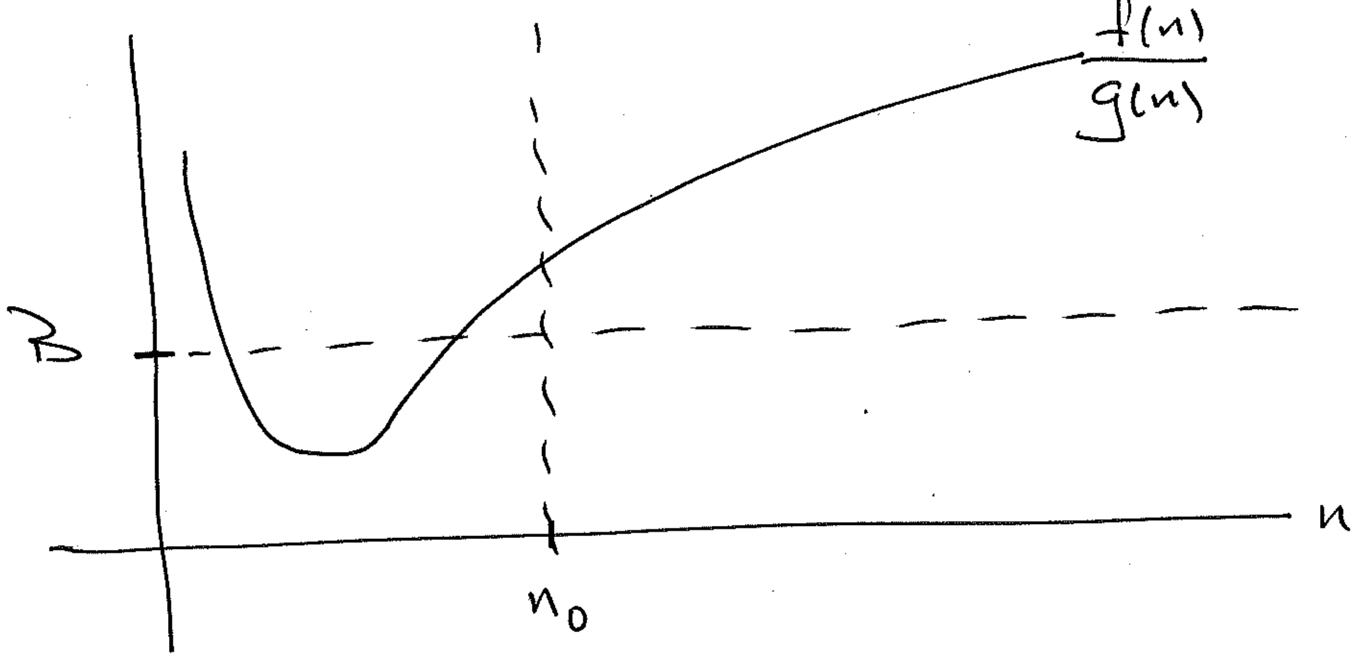
<u>Defn</u>

$f(n) = \Omega(g(n))$ iff there exists $B > 0$ and $n_0 > 0$ such that:

$$B \leq \frac{f(n)}{g(n)}$$

for all $n \geq n_0$. We say $g(n)$ is an <u>asymptotic lower bound</u> for $f(n)$.

<u>meaning</u>: $f(n)$ grows no slower than $g(n)$




<u>Ex.</u> $f(n) = 6n^3 + 4n$, $g(n) = 2n^2$. <u>claim</u>:

$6n^3 + 4n = \Omega(2n^2)$. check:

$$\frac{f(n)}{g(n)} = 3n + \frac{2}{n} \geq 7 \text{ for all } n \geq 2$$

$\uparrow$ B $\qquad \uparrow$ $n_0$

<u>Generalize (exercise)</u>:

$an^3 + bn = \Omega(cn^2)$ for any $a, b, c \in \mathbb{R}$

$a > 0$, $c > 0$

observe:

(1) $n \geq \cancel{n_1} \Rightarrow \frac{f(n)}{g(n)} \leq B_1 \quad \left(f(n) = O(g(n))\right)$

$n_0$

and

(2) $n \geq \cancel{n_2} \Rightarrow B_2 \leq \frac{g(n)}{f(n)} \quad \left(g(n) = \Omega(f(n))\right)$

$n_0$

are equivalent (logically).

let $n_0 = \max(n_1, n_2)$.

let $B_2 = \frac{1}{B_1}$

Thus: $f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

Analogy: $X \leq Y$ iff $Y \geq X$

<u>Defn</u>

$f(n) = \Theta(g(n))$ iff there exist positive $B_1, B_2, n_0$ such that

$$B_1 \leq \frac{f(n)}{g(n)} \leq B_2$$

for all $n \geq n_0$.

We say: $g(n)$ is a <u>tight asymptotic bound</u> for $f(n)$

meaning: $f(n)$ and $g(n)$ have the same asymptotic growth rate.

<u>note</u>: its obvious that

$$f(n) = \Theta(g(n)) \text{ iff both } \begin{cases} f(n) = O(g(n)) \\ \text{and} \\ f(n) = \Omega(g(n)) \end{cases}$$

<u>Analogy</u>

$f(n) = O(g(n)) \quad \sim \quad X \leq Y$

$f(n) = \Omega(g(n)) \quad \sim \quad X \geq Y$

$f(n) = \Theta(g(n)) \quad \sim \quad X = Y$

<u>Theorem</u>

$f(n) = \Theta(g(n))$ iff $g(n) = \Theta(f(n))$.

<u>Ex.</u> $f(n) = 5n^2 + 11n - 24$, $g(n) = n^2$.

<u>claim</u>: $5n^2 + 11n - 24 = \Theta(n^2)$.

True since

$$\underset{B_1}{4} \le \frac{5n^2 + 11n - 24}{n^2} \le \underset{B_2}{6}$$

for all $n \ge \underset{n_0}{8}$.

<u>Defn</u>
$f(n) = o(g(n))$ iff $\lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right) = 0$.

we say $g(n)$ is a <u>strict asymptotic upper bound</u> for $f(n)$.

meaning: $f(n)$ grows strictly slower than $g(n)$.

Ex.

$f(n) = \ln(n)$, $g(n) = n$

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty}\left(\frac{\ln(n)}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1/n}{1}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n}\right) = 0$$

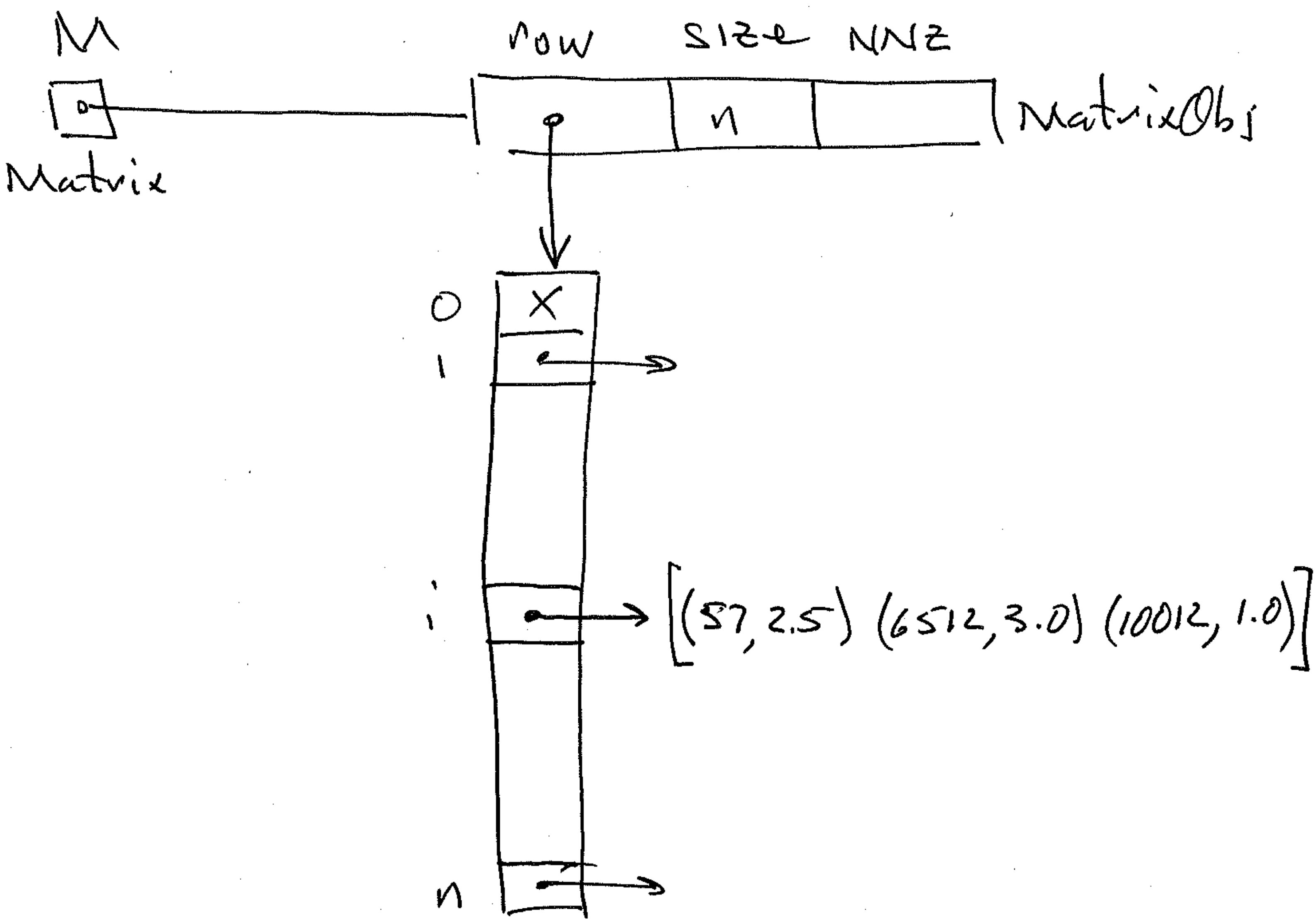
$\therefore\ \ln(n) = o(n)$

Generalize: $\quad (\log_b(n))^m = o(n^k)$

for any $b > 1$, $m \geq 0$, $k \geq 0$.

Pa 4 :

Matrix ADT




Pair