CSE 101 2-13-24

Recall:

(1) $f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

Similarly

(2) $f(n) = o(g(n))$ iff $g(n) = \omega(f(n))$

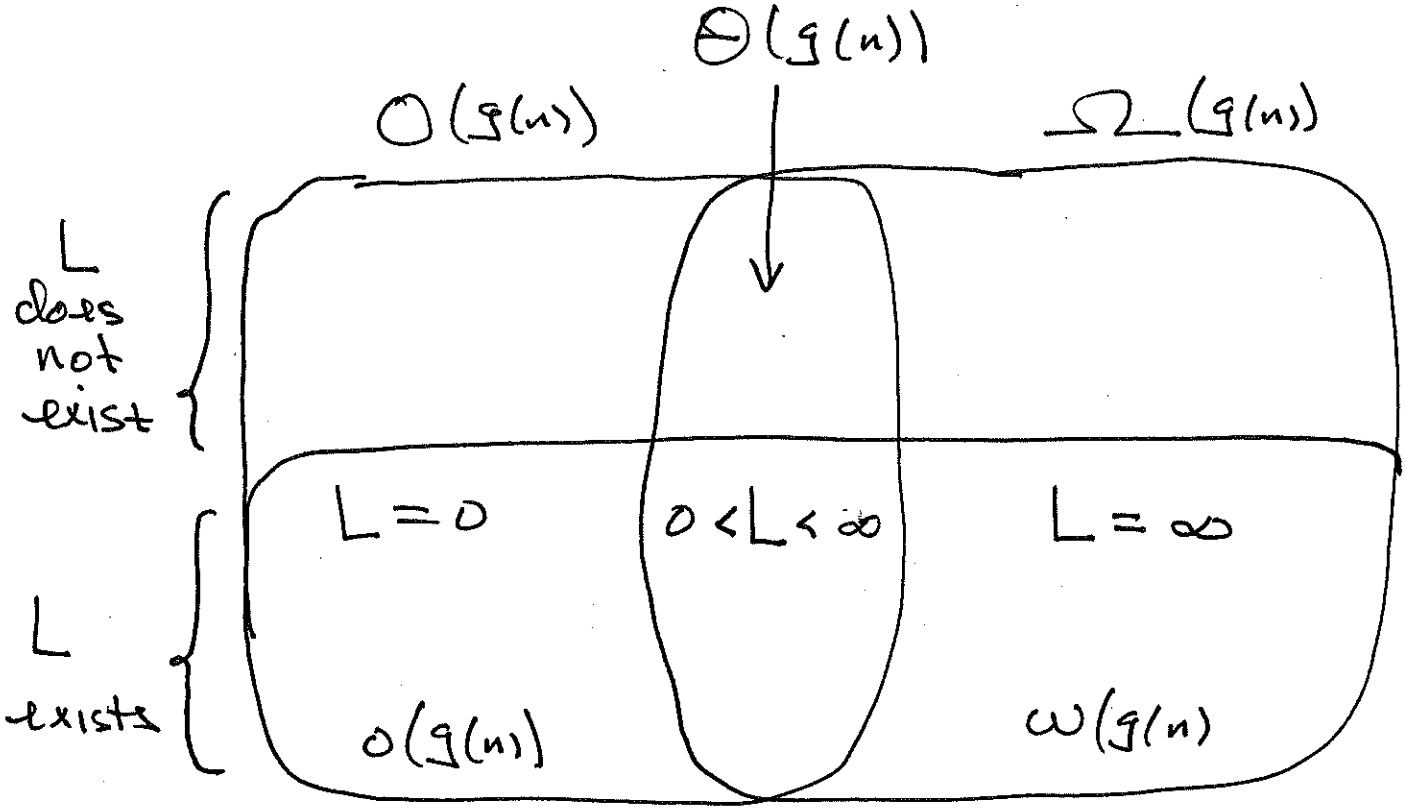
analogy:

(1)': $x \le y$ iff $y \ge x$

(2)': $x < y$ iff $y > x$

Picture:

let $L = \lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right)$, if it exists



Thm if $\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = L$ then

- $L = 0 \iff f(n) = o(g(n)) \overset{\not\Leftarrow}{\Rightarrow} f(n) = O(g(n))$
- $L = \infty \iff f(n) = \omega(g(n)) \overset{\not\Leftarrow}{\Rightarrow} f(n) = \Omega(g(n))$
- $0 < L < \infty \overset{\not\Leftarrow}{\Rightarrow} f(n) = \Theta(g(n))$

note! converses are false

Exercises

(3) let $c > 0$. show $c \cdot f(n) = O(f(n))$

Proof:

$$\frac{c \cdot \cancel{f(n)}}{\cancel{f(n)}} = c \leq B \text{ for all } n \geq n_0$$

let $B = c$, $n_0 = 1$ ∎

(4) $\log_b(n) = \Theta(\log_a(n))$ for any $a > 1, b > 1$.

Proof

$$\log_b(n) = \frac{\ln(n)}{\ln(b)} \text{ and } \log_a(n) = \frac{\ln(n)}{\ln(a)}$$

$$\therefore \frac{\log_a(n)}{\log_b(n)} = \frac{\cancel{\ln(n)}/\ln(a)}{\cancel{\ln(n)}/\ln(b)} = \frac{\ln(b)}{\ln(a)} = \text{const.} \in (0, \infty)$$

$\therefore \log_a(n) = \Theta(\log_b(n))$. ∎

(6) $f(n) + o(f(n)) = \Theta(f(n))$ ✓

Note: $o(f(n))$ stands for some fcn. $h(n)$ s.t. $h(n) = o(f(n))$.

Proof

We have $\frac{h(n)}{f(n)} \to 0$. So

$$\frac{f(n)+h(n)}{f(n)} = 1 + \underbrace{\frac{h(n)}{f(n)}}_{\to 0} \to 1 \in (0,\infty)$$

$\therefore$ $f(n) + h(n) = \Theta(f(n))$ ∎

Selection_Sort(A)

```
n = length[A]
for i = 1 to n-1
    imin = i
    for j = i+1 to n
        if A[j] < A[imin]
            imin = j
    A[i] <-> A[imin]   // swap
```

$$\underbrace{A_1\ A_2\ \cdots\cdots A_{i-1}}_{\text{sorted}}\ \underbrace{A_i\ A_{i+1}\ \cdots\ A_n}_{\text{unsorted}}$$

$(i-1)$ smallest elements in A

<u>Basic op</u>: comparison of 2 array elements: line 5

$$\#\text{comp} = (n-1)+(n-2)+\cdots+3+2+1$$

$$= \frac{(n-1)((n-1)+1)}{2} = \frac{n(n-1)}{2}$$

$$= \frac{1}{2}n^2 - \frac{1}{2}n = \Theta(n^2)$$

<u>note</u>: $an^2+bn+c = \Theta(n^2)$ for any $a>0$, $b,c \in \mathbb{R}$. why

$$\frac{an^2+bn+c}{n^2} = a + \underset{\downarrow \atop 0}{\frac{b}{n}} + \underset{\downarrow \atop 0}{\frac{c}{n^2}} \rightarrow a \in (0,\infty)$$

$\therefore$ $an^2+bn+c = \Theta(n^2)$

- BFS:

let $n = |V(G)|$, $m = |E(G)|$.

- initialize: $\Theta(n)$
- queue ops: $\Theta(n)$ (worst case)
- Process adj lists: Θ(total len. of all adj lists)

$$= \Theta(m)$$

$$\text{total cost} = \Theta(n) + \Theta(n) + \Theta(m)$$

$$= \boxed{\Theta(\underbrace{n+m})}$$

$$= \Theta(\text{const.} \cdot (\#\text{bytes needed}))$$

- DFS : $\Theta(n+m)$

Where $n = |V(G)|$, $m = |E(G)|$.

Exercise:

- $\Theta(f(n) + g(n)) = \Theta(f(n)) + \Theta(g(n))$
- $\Theta(f(n) \cdot g(n)) = \Theta(f(n)) \cdot \Theta(g(n))$.