
• mid1 on Thur.

To find the SCCs of a digraph G , do

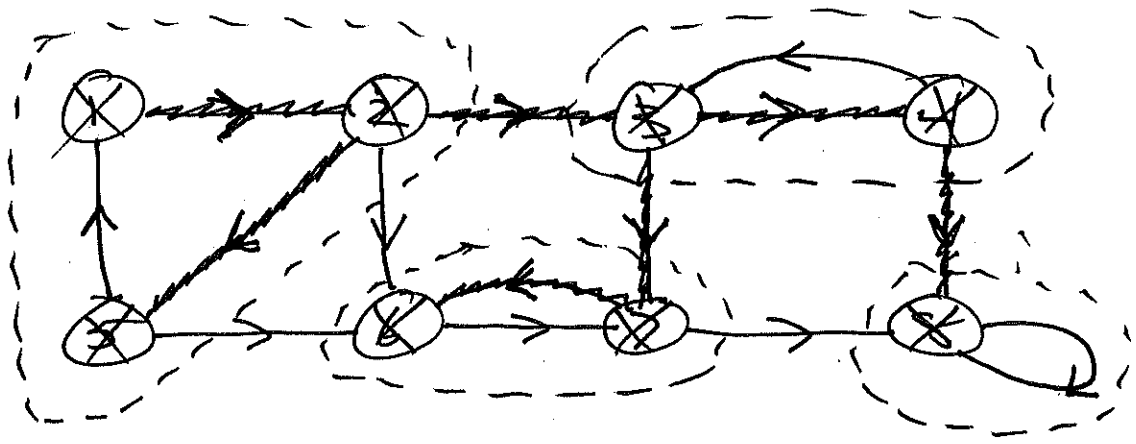
- Run DFS(G), as vertices finish, push onto a stack.
- Compute G^T , the transpose of G
- Run DFS(G^T), process vertices in main loop of DFS

Theorem

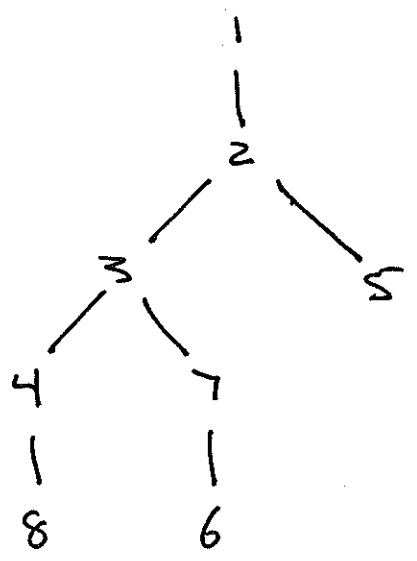
when this is done, the trees in the DFS forest from 2nd call to DFS have vertex sets the SCCs of G.

Ex

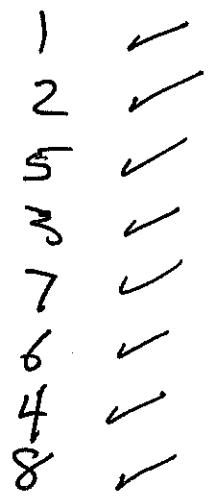
G



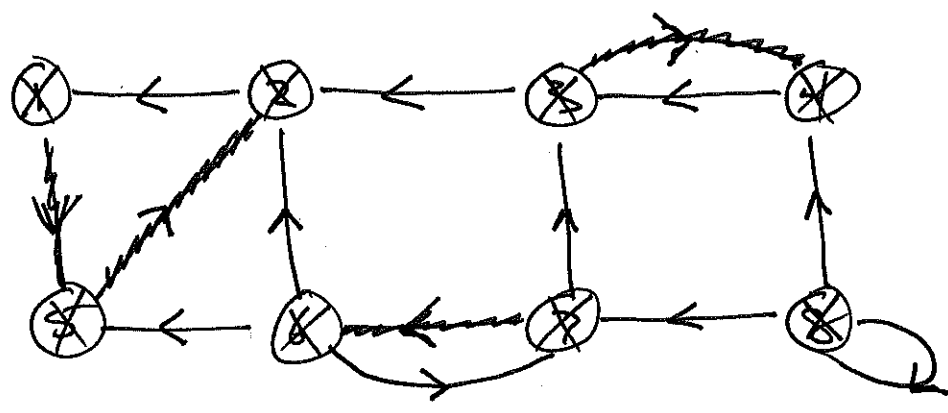
Forest



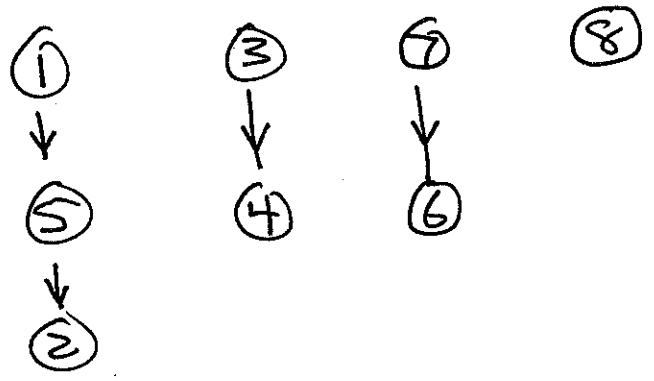
Stack



G^T



forest



	<u>stack</u>	<u>Parent</u>
C_4	8	n
C_3	7, 6	n, -
C_2	3, 4	n, -
C_1	1, 5, 2	n, -, -

- SCCs:
- $\{1, 5, 2\} = C_1$
 - $\{3, 4\} = C_2$
 - $\{7, 6\} = C_3$
 - $\{8\} = C_4$

Data

The component Graph (also Condensation graph) of G , denoted G^{SCC} , is

the digraph with

$$V(G^{\text{SCC}}) = \{ C_1, C_2, \dots, C_k \}$$

SCCs of G

$$E(G^{\text{SCC}}) = \{ \dots (C_i, C_j) \dots \}$$

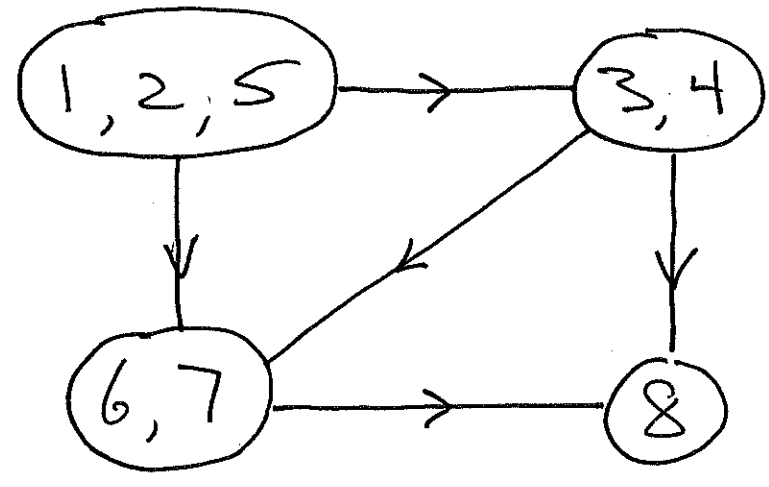
where $(C_i, C_j) \in E(G^{\text{SCC}})$ iff

there exist $x \in C_i, y \in C_j$ with

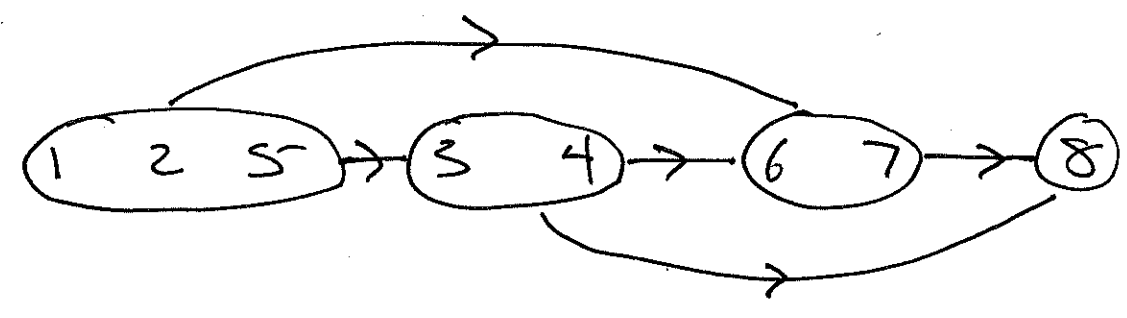
$$(x, y) \in E(G)$$

Ex.

G^{sec}



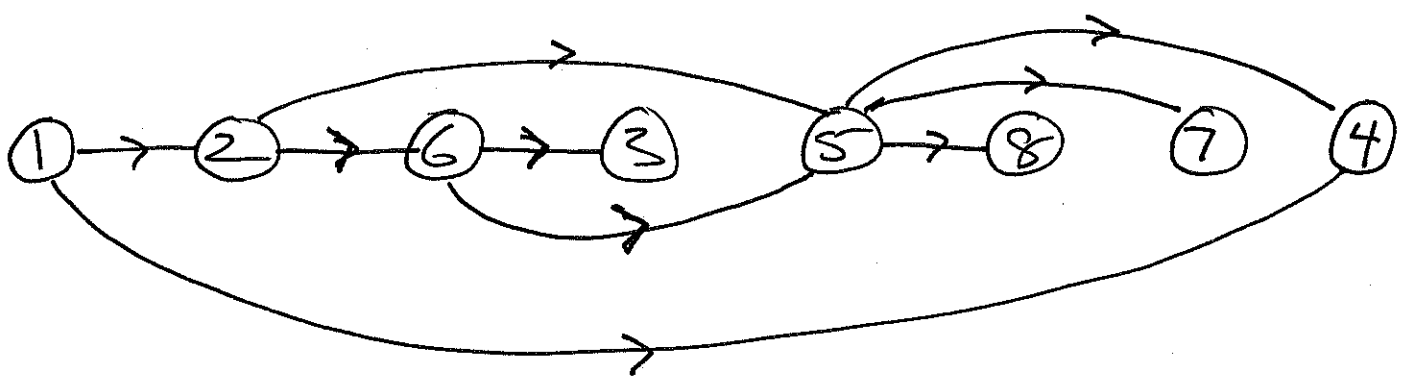
Topological sort!



note:

$$(G^T)^{sec} = (G^{sec})^T$$

Problem 5 from Review Sheet



#(permutations of {8, 7, 4}) = 3! = 6

#(positions for 3) = 5

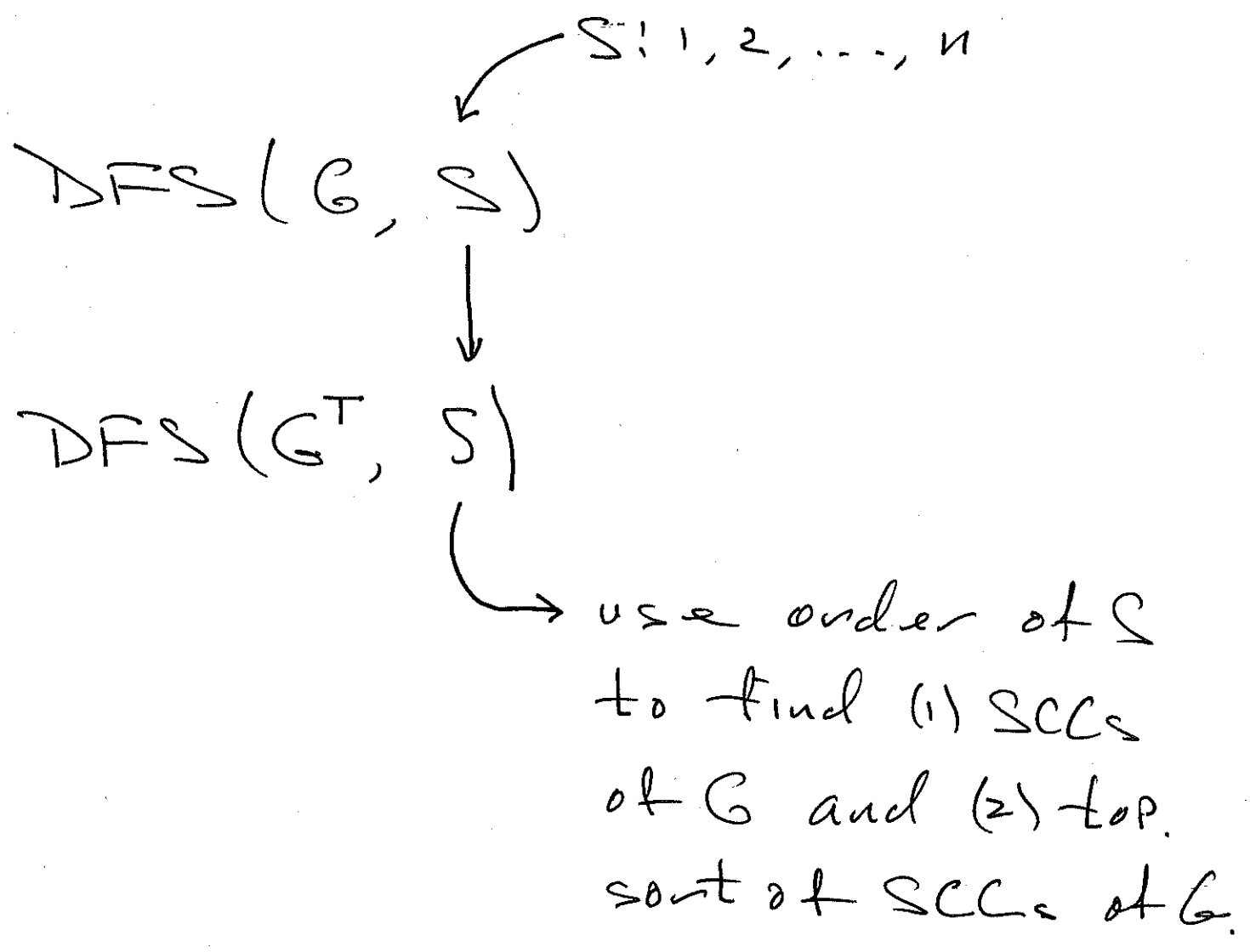
#topological sorts = 6 · 5 = 30

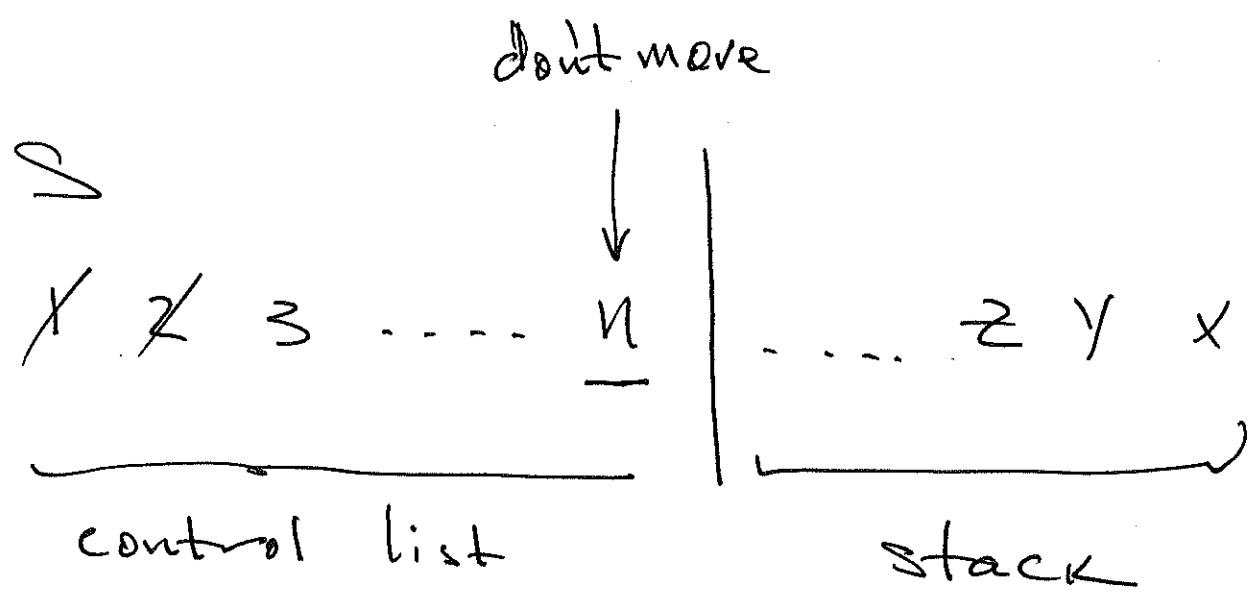
Part 1:

DFS new Prototype

void DFS (Graph G, List S)

Call like this:





Push = insertAfter()