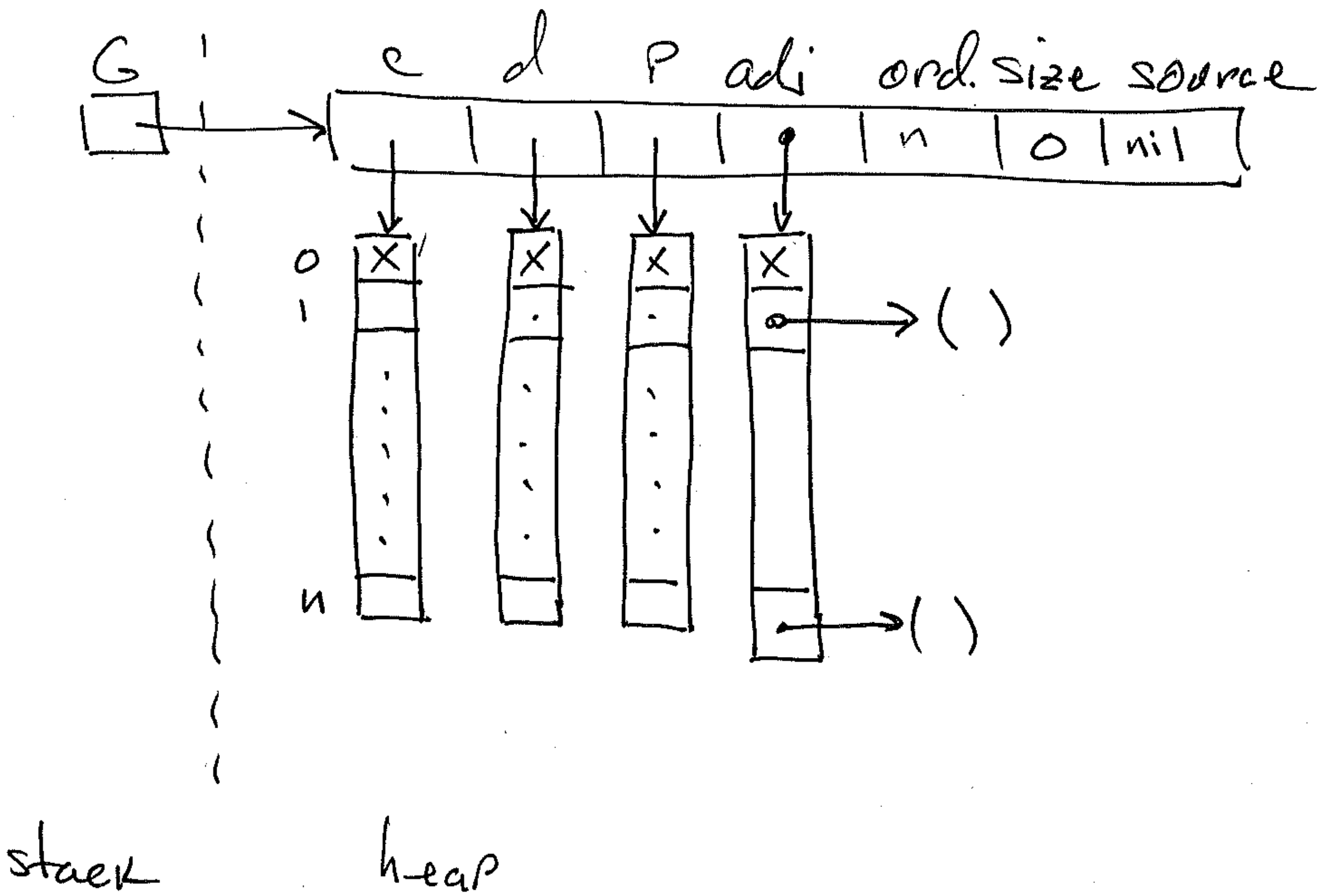
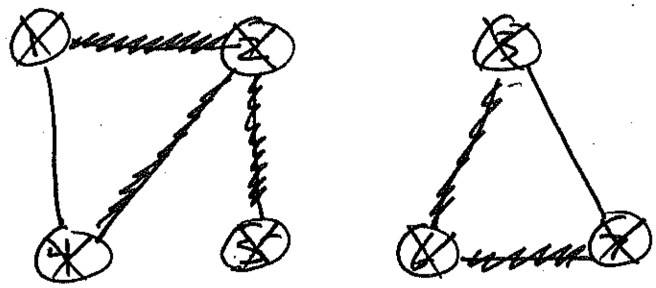
cse 101 1-25-24

- midterm 1 : Thur February 1
- Pa2 : ext. 2 days Sun. 10 Pm.

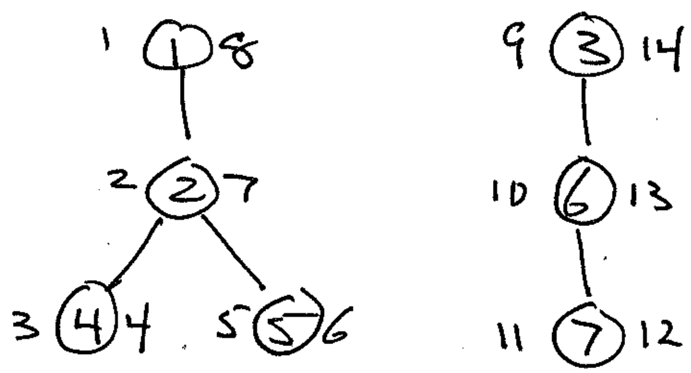
Graph ADT



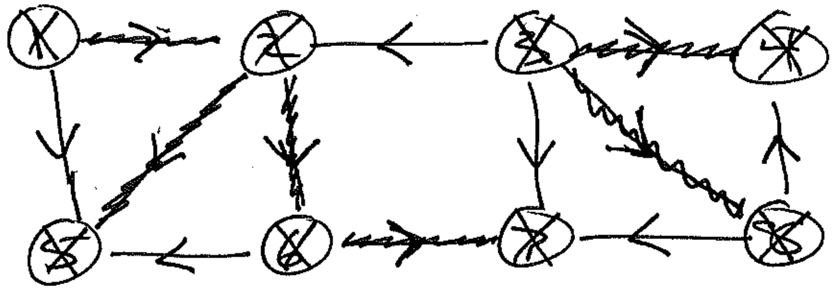
Ex



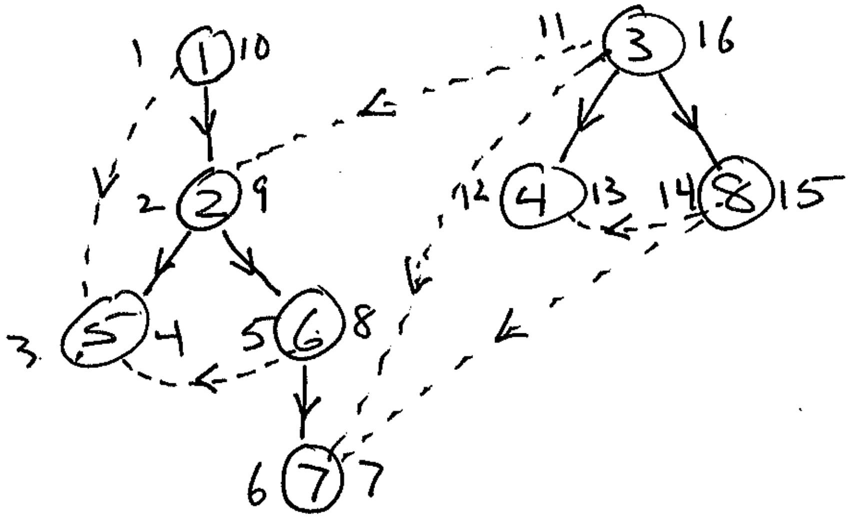
DFS Forest:



Ex.



DFS forest



Predecessor subgraph : (V_p, E_p)

$$V_p = V(G),\ E_p = \{ \underbrace{(P[x], x)} \mid P[x] \neq nil \}$$

ordered pair if dir.
unordered pair if undir.

Edge Classification

- Tree: belong to $E_p = E(G_p)$
- Back: join descendant to ancestor
- Forward: join ancestor to descendant (other than child)
- cross: • cousin to cousin in same tree
  • tree to tree

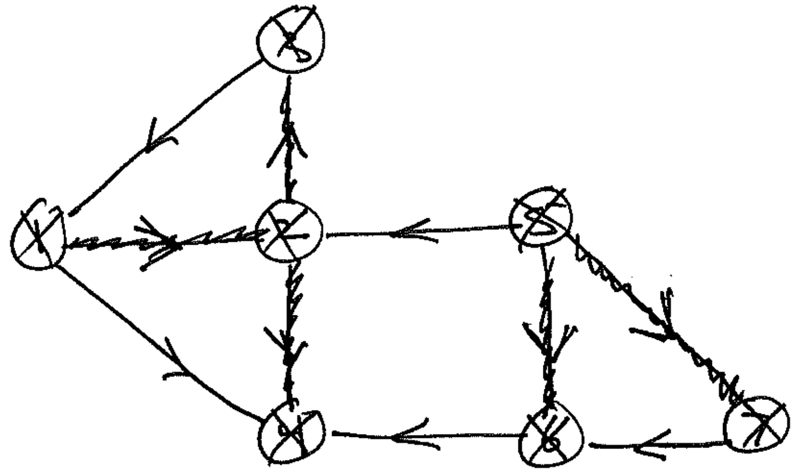
Ex. tree: (1, 2), (2, 5), (2, 6), (6, 7), (3, 4), (3, 8)

back:

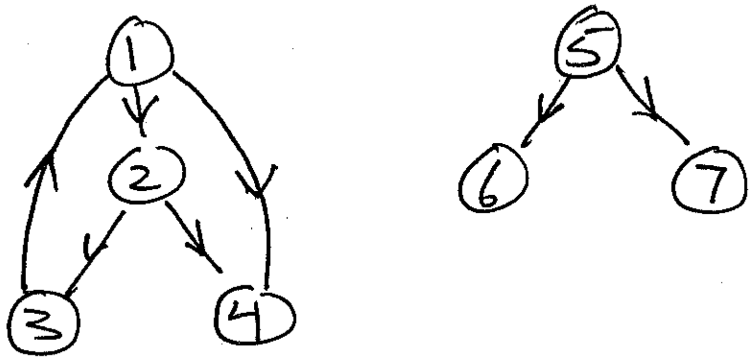
forward: (1, 5)

cross: (6, 5), (3, 2), (3, 7), (8, 7), (8, 4)

Ex



Forest



tree: (1, 2), (2, 3), (2, 4), (5, 6), (5, 7)

back: (3, 1)

forward: (1, 4)

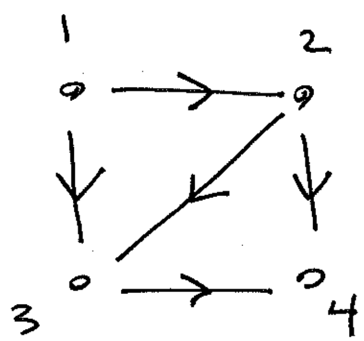
cross: (5, 2), (6, 4), (7, 6)

Topological Sort

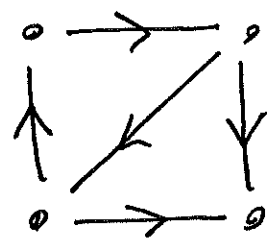
A digraph $G = (V, E)$ is called <u>acyclic</u> (also a DAG) iff it contains no directed cycles.

<u>cycles</u>:

<u>Ex.</u>



<u>acyclic</u>

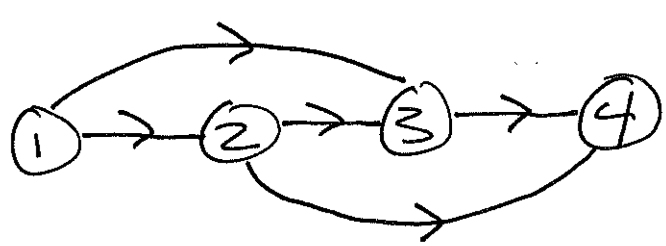


<u>not</u> acyclic

Defn

Let $G = (V, E)$ be a DAG. A topological sort of V is a linear ordering of V such that if $(x, y) \in E$, then x must appear before y in the ordering.

Ex. (Previous ex.)

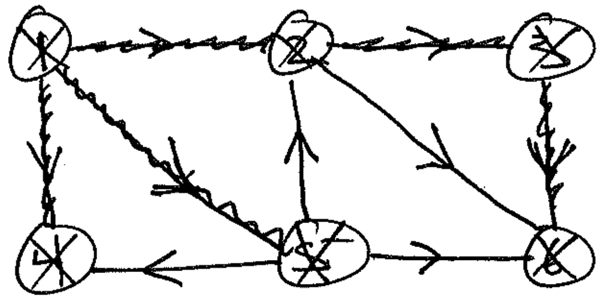


Ex.

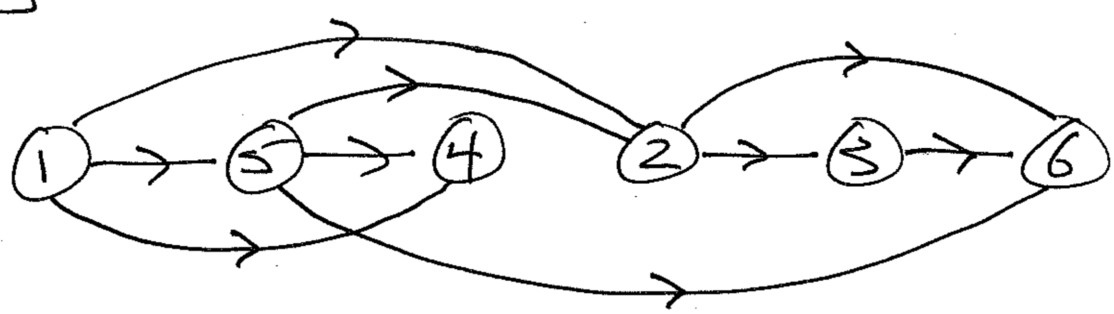
stack

1
5
4
2
3
6

G



topological sort:



To find a topological sort:

- Run DFS (G)
- as vertices finish, push onto stack

when done, stack is top. sort.

Strongly Connected Components

Let $G=(V,E)$ be a digraph.

Defn We say $y\in V$ is reachable from $x\in V$ iff G contains a directed x-y path.

Defn We say G is strongly connected iff for all $x,y\in V$, Both y is reachable from x and x is reachable from y.

More generally, we say

$$U \subseteq V(G)$$

is strongly connected iff for all $x, y \in U$, Both y is reachable from x, and x from y.

Defn
We say $U \subseteq V$ is a strongly connected component (SCC) of G iff

(1) U is strongly connected

and (2) U is maximal w.r.t. (1)

(2) means that if

$$U \subsetneq W \subseteq V$$

then, W is <u>not</u> strongly connected.

Ex