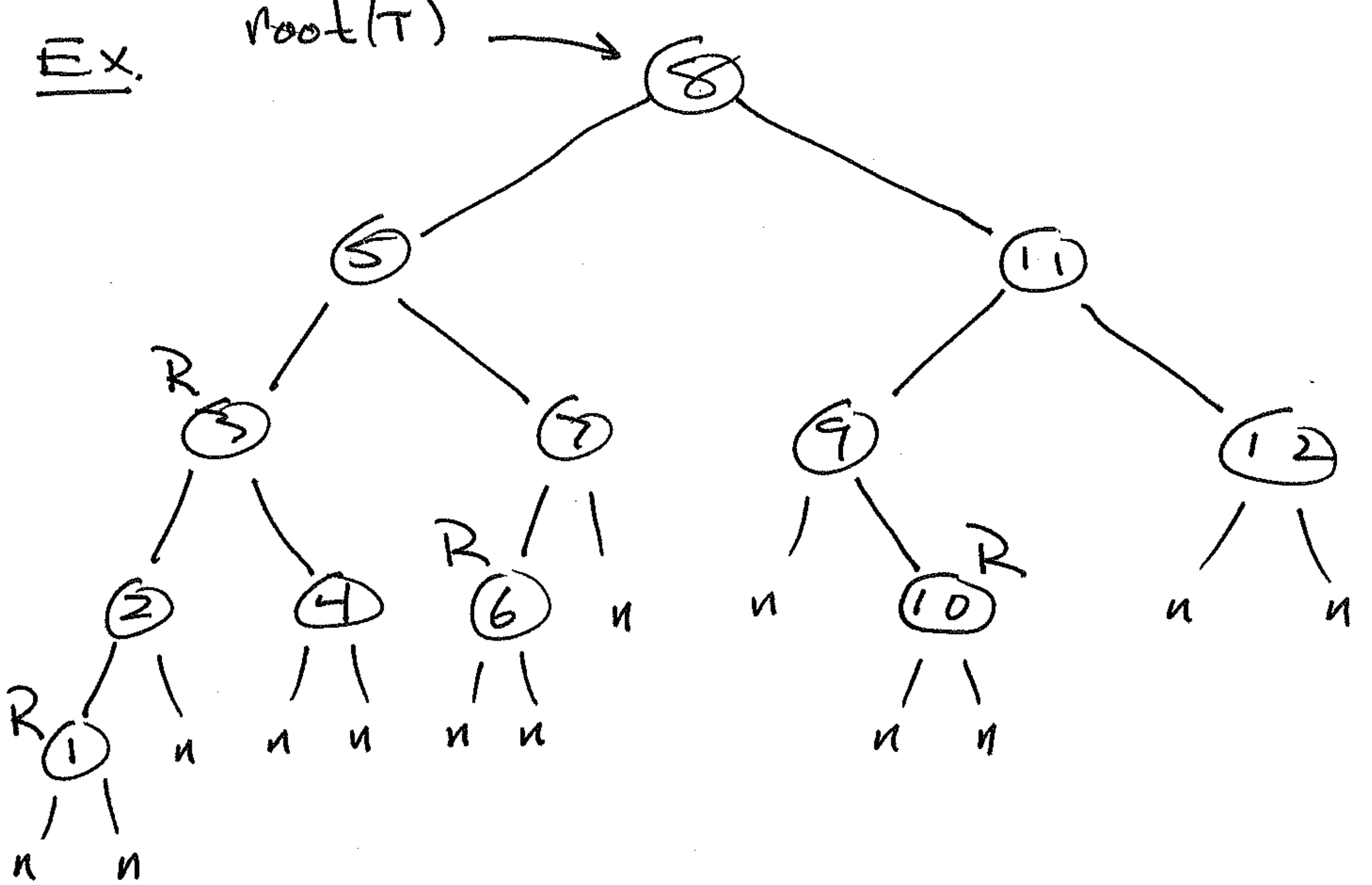
CSE 101-02 5-24-22

- mid 2 solns posted.
- Pa8 " posted
- Pa7: ext. last time (Wed 10PM).

## RBTs ... continue

notes: should say RBT properties.

Ex.




| node | black height |
|---|---|
| 8 | 3 |
| 5, 11, 3 | 2 |
| 1, 2, 4, 6, 7, 9, 10, 12 | 1 |
| nil leaves | 0 |

note: $bh(x) = 0$ iff $height(x) = 0$ iff $x$ is leaf (nil)

Theorem

An RBT with $n$ internal (non-nil) nodes and height $h$ satisfies

$$h \leq 2 \lg(n+1).$$

Rmks

- any BST satisfies $h \geq \lfloor \lg n \rfloor$

  so an RBT satisfies

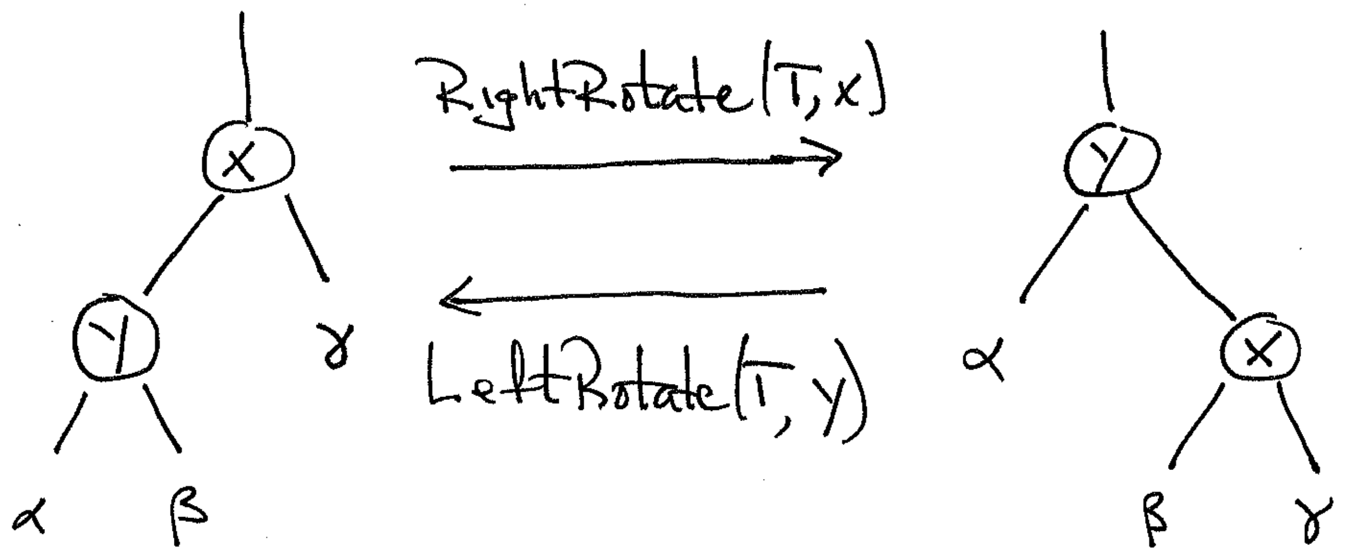
  $$\Omega(\log n) \leq \text{height}(T) \leq O(\log n)$$

  so: $\text{height}(T) = \Theta(\log n)$.

- Queries run in time $\Theta(\log n)$.

## 13.2 Rotations

In Pictures




$\alpha, \beta, \gamma$ are arbitrary subtrees.

Preserve BST Properties:

$$\text{keys}(\alpha) \leq \text{key}[y] \leq \text{keys}(\beta) \leq \text{key}[x] \leq \text{keys}(\gamma)$$

Summary of RightRotate$(T, x)$

$$\beta \begin{cases} \circ \ \mathrm{left}[x] = \mathrm{right}[y] \\ \circ \ p[\mathrm{right}[y]] = x \end{cases}$$

$$\text{Parent} \begin{cases} \circ \ P[y] = P[x] \\ \circ \ P[x]\text{'s (left or right) child} = y \end{cases}$$

$$x \Leftrightarrow y \begin{cases} \circ \ \mathrm{right}[y] = x \\ \circ \ P[x] = y \end{cases}$$

see: Pseudo-code

## 13.3 insertion

- color new node Red
- do BST insert
- fix-up RBT properties.

Pseudo-code

RB_Insert(T, z)

RB_InsertFixUP(T, z)

RB-Insert Fixup:

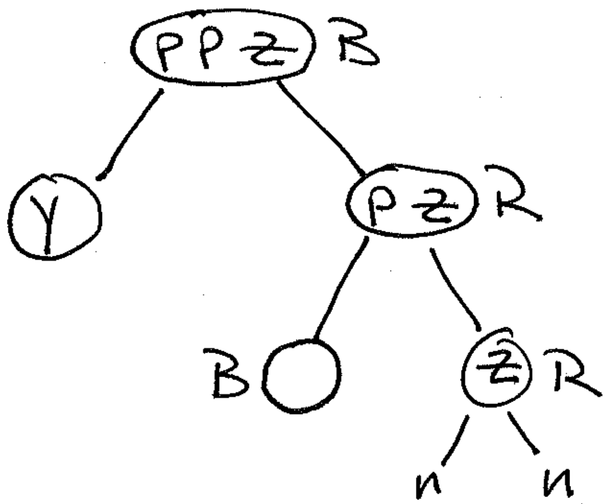
Book does cases 1, 2, 3.

we do cases 4, 5 6

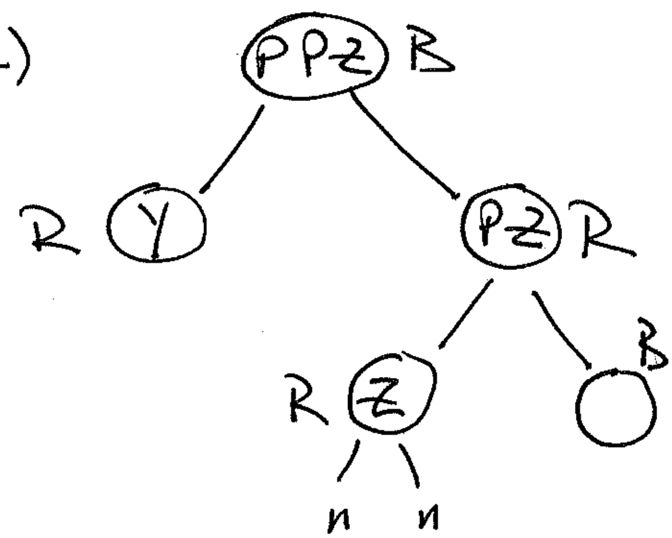
set $y = left[p[p[z]]]$ z's uncle

Case 4: y is R

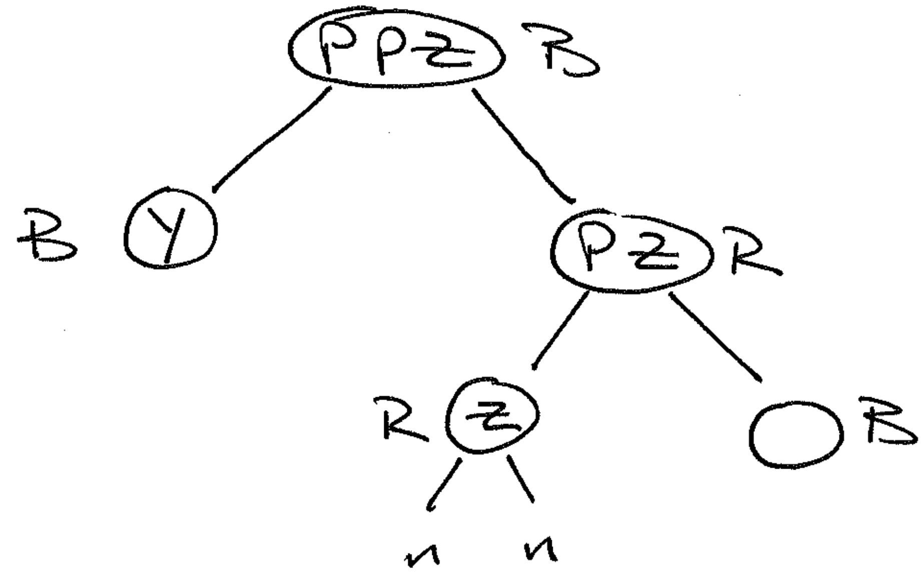
(4.1)



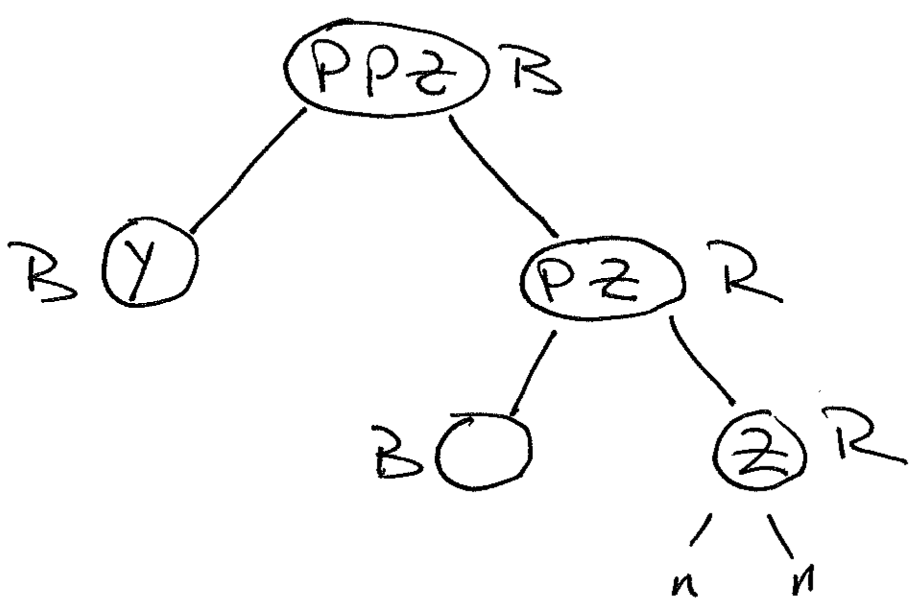

(4.2)



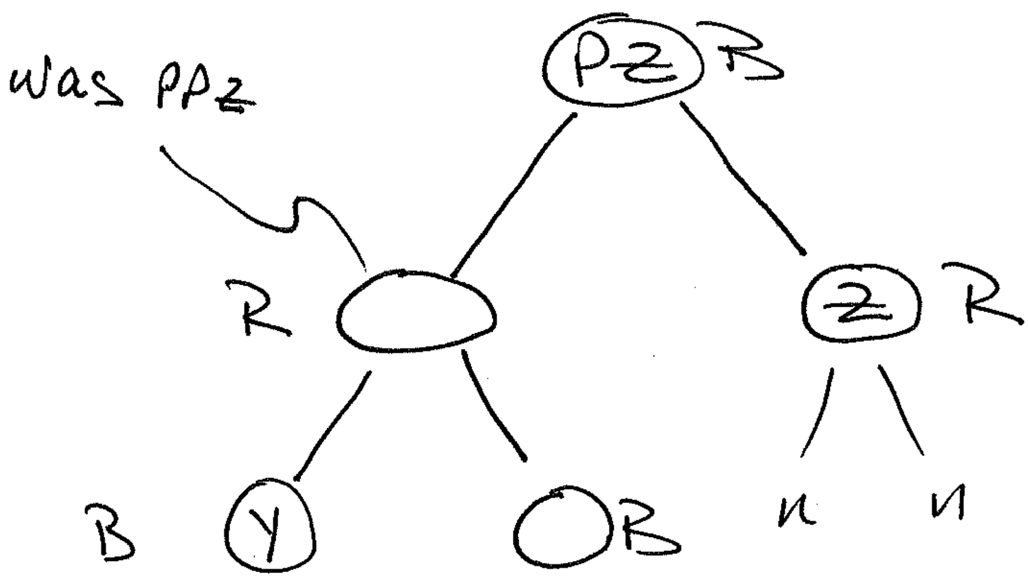
Case 5: y is Black, z is left[p[z]]




Case 6: y is Black, z is right[p[z]]

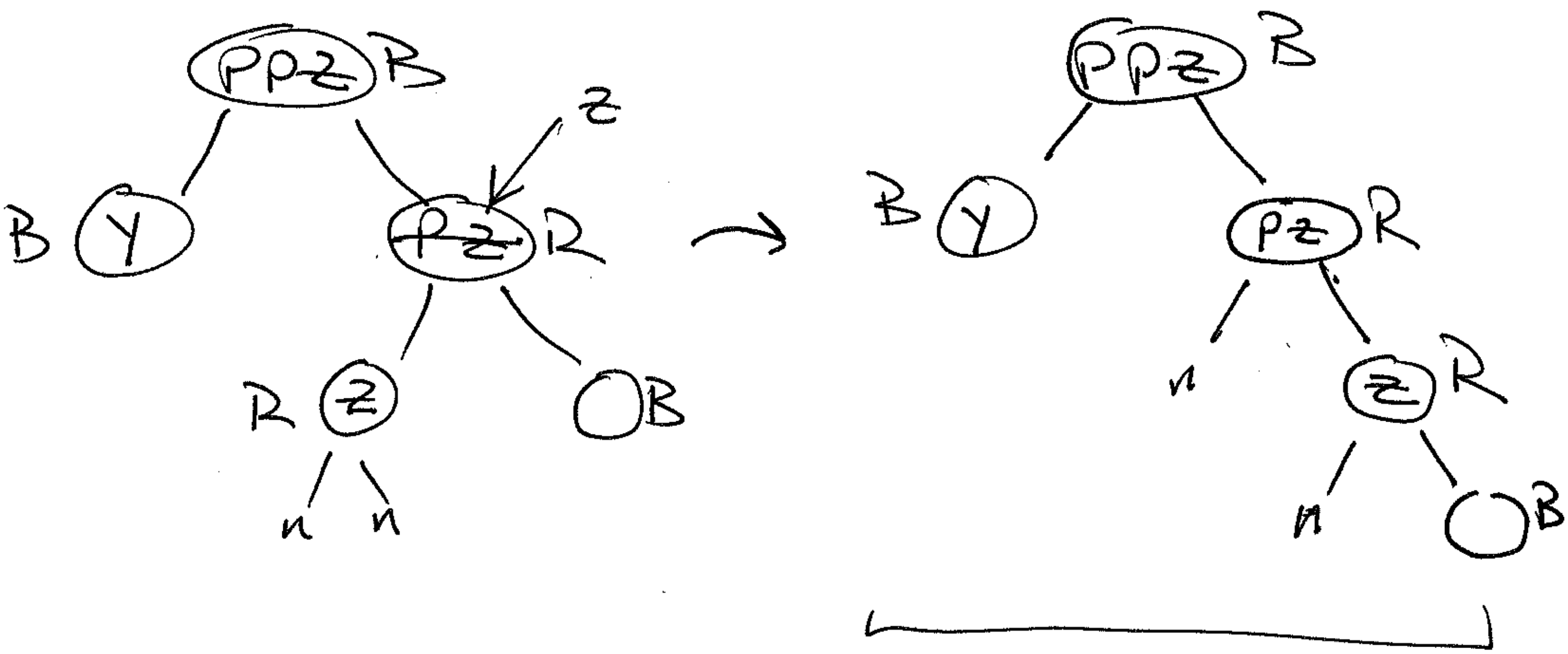


In case 6: color PZ Black, color PPZ Red, then left rotate about PPZ



In case 5: convert to case 6

by letting local var z 'climb up' to pz, then right rotate about z

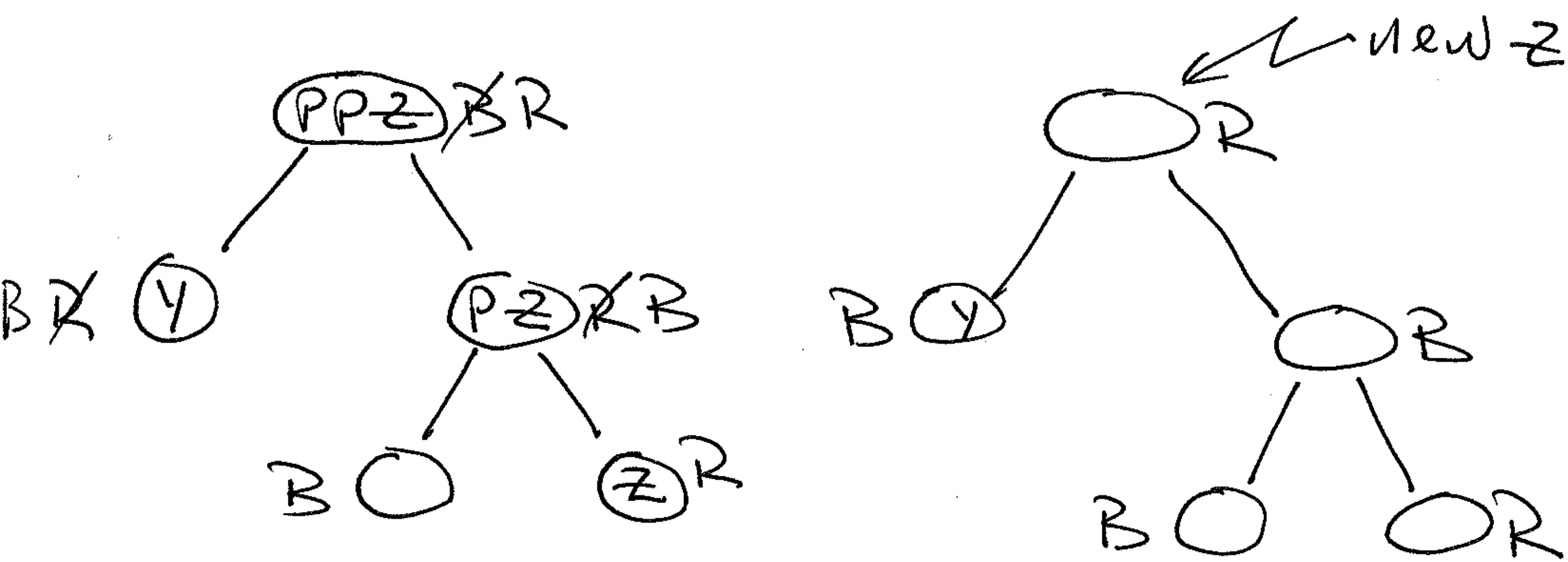



now case 6

In case 4:

fix color relations between y, pz, and ppz, then let z 'climb up' to ppz

(4.1)

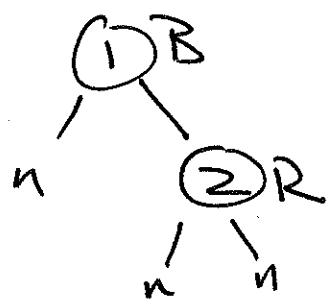



may have another iteration of whole loop.

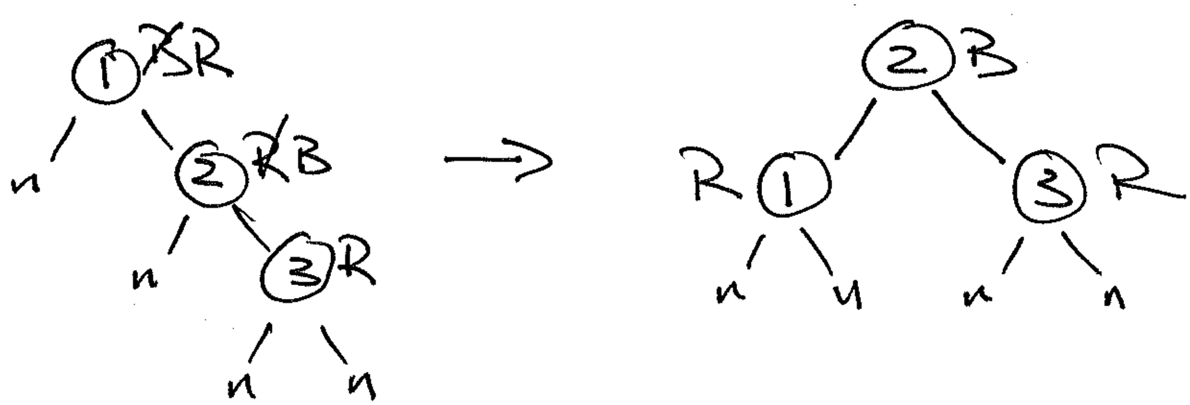
Ex. insert : ~~1~~, ~~2~~, ~~3~~, ~~4~~, 5 into an initially empty RBT.

insert(1)


insert(2)

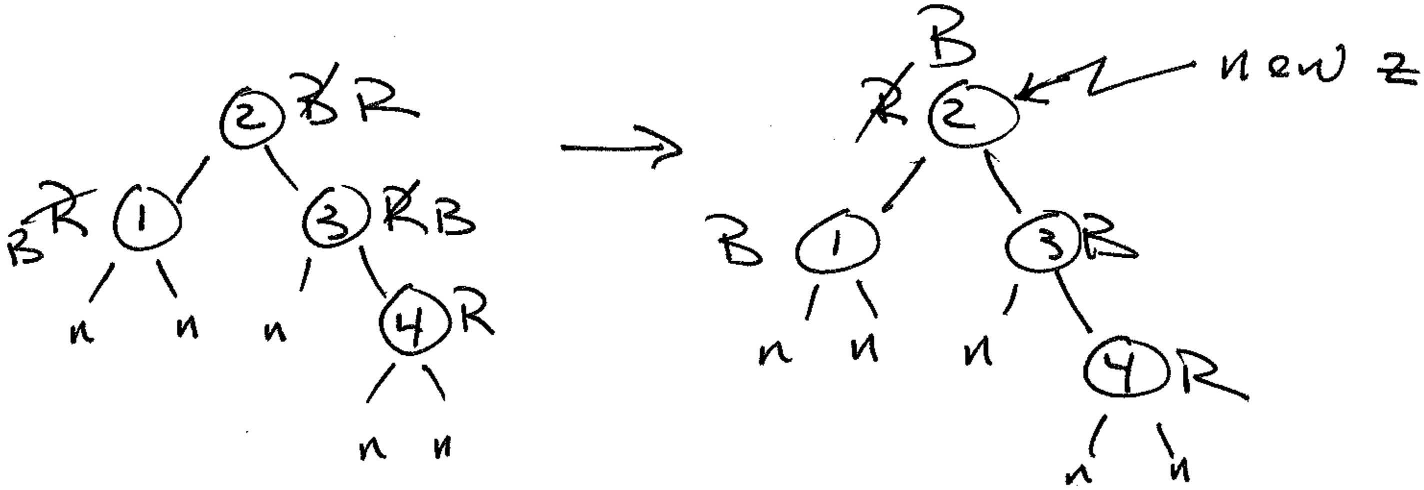



insert(3)




(case 6)

insert(4)




(case 4)

insert(5)