

Pass: one last day ext. (Sat. 10pm)

continue . . .

### lemma 2

After  $\text{Initialize}(G, s)$ , the inequality

$$\delta(s, x) \leq d[x] \quad (\forall x \in V(G))$$

is maintained over any sequence of relaxations.

### Lemma 3 (Path Relaxation Property)

$\Rightarrow$

$P: s = x_0, x_1, x_2, \dots, x_k = y$

is a min-weight  $s$ - $y$  path,  
and if the edges of  $P$  are  
relaxed in order

$(x_0, x_1), (x_1, x_2), \dots, (x_{k-1}, x_k)$

then  $d[y] = \delta(s, y)$ . This is true  
regardless of any other relaxations  
interspersed with the above.

BellmanFord:

Runtime:  $n = |V|$ ,  $m = |E|$

Initialize:

$$\text{cost} = \Theta(n)$$

Each call to  $\text{Relax}(\cdot, \cdot)$  costs:  $\Theta(1)$

• 1<sup>st</sup> loop :-

$$\text{cost} = \Theta((n-1)m) = \Theta(nm)$$

• 2<sup>nd</sup> loop

$$\text{cost} = \Theta(m)$$

$$\text{total cost} = \Theta(n) + \Theta(nm) + \Theta(m)$$

$$= \Theta(nm)$$

# Dijkstra: Runtime

$$n = |V|, m = |E|$$

• ExtractMin :  $\Theta(\log n)$

total calls to ExtractMin :  $\Theta(n \log n)$

• Relax :  $\Theta(\log n)$

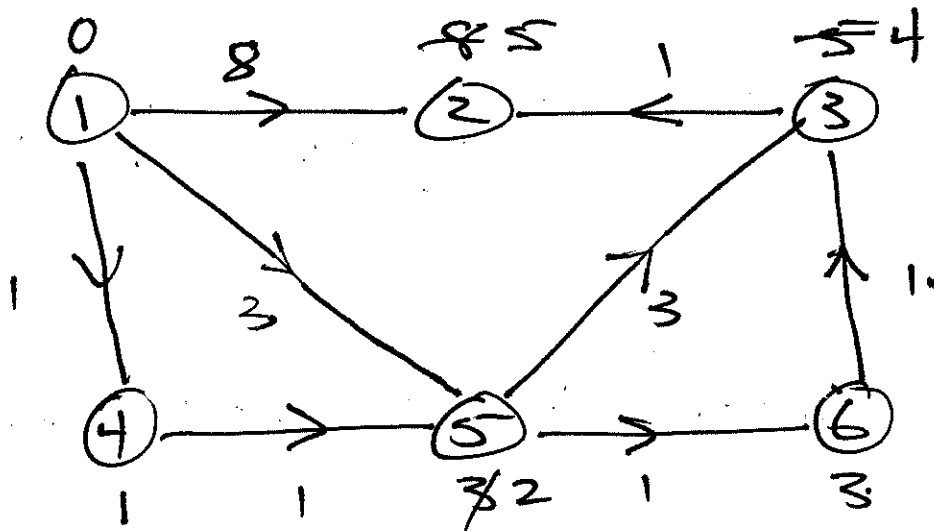
total calls to Relax :  $\Theta(m \log n)$

total cost of Dijkstra:

$$\Theta(n \log n) + \Theta(m \log n)$$

$$= \Theta((n+m) \log n)$$

Ex.  $S=1$



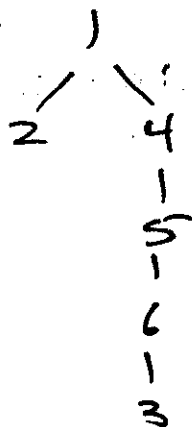
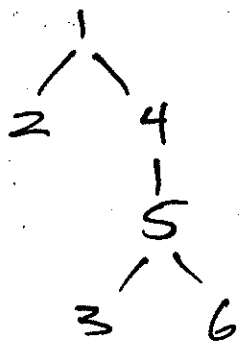
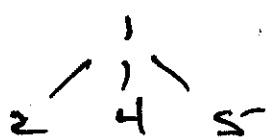
PQ: ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~

Q: 0 ~~5~~ ~~4~~ 1 ~~2~~ 3

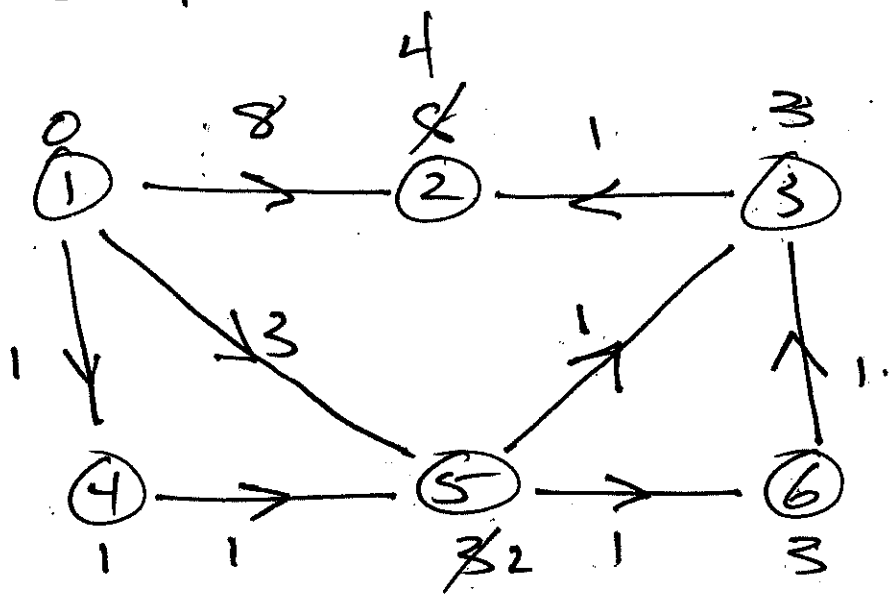
P: 11 ~~3~~ ~~6~~ 1 ~~4~~ 5

Remove Order! 1 4 5 6 3 2

Tree!



Ex.  $s=1$



PQ: ~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ ~~6~~  
 d: 0 ~~4~~ 3 1 ~~2~~ 3  
 p: 1 ~~3~~ 5 1 ~~4~~ 5

Remove Order: 1 4 5 3 6 2

Tree:

