cse 101-02 2-9-23

Pa4: Matrix Operations

Build order

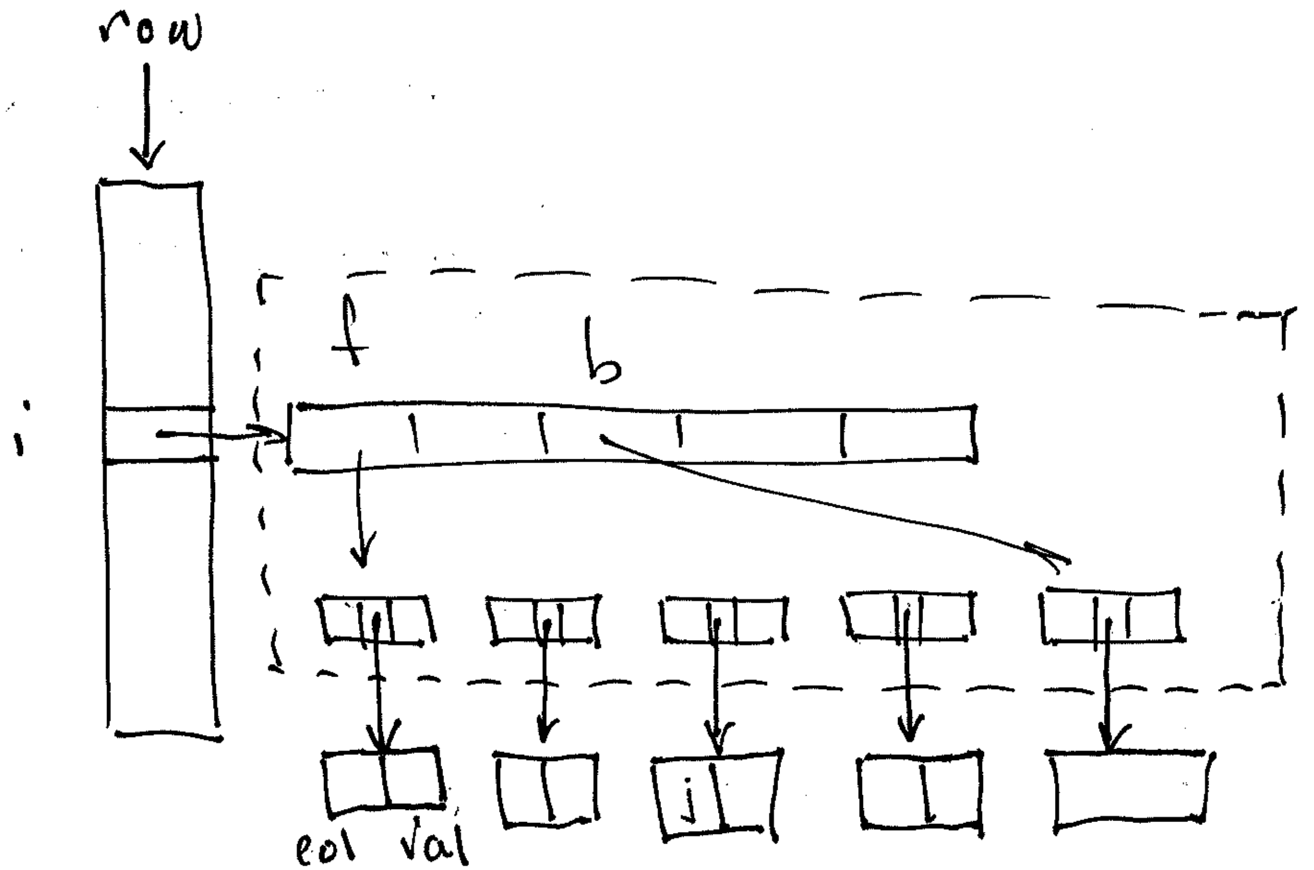
- constructor
- makeZero()
- destructor
- changeEntry() ←
- PrintMatrix()

  ⋮

- copy()
- transpose()
- scalarMult()

changeEntry(M, i, j, x)

1. $x = 0$, $M_{ij} = 0$: nothing ✓
2. $x = 0$, $M_{ij} \neq 0$: delete ✓
3. $x \neq 0$, $M_{ij} = 0$: insert ✓
4. $x \neq 0$, $M_{ij} \neq 0$: overwrite ✓



product()
sum()
diff()

$$(A \cdot B)_{ij} = (\text{row } i \text{ of } A) \bullet (\text{col } j \text{ of } B)$$

$$= (\text{row } i \text{ of } A) \bullet (\text{row } j \text{ of } B^T)$$

helper function: dot()

heading

double dot(List L, List P)

row i of A   row j of B^T

L : (10,) (30) (50, •) (60,)

P : (20,) (40,) (50, •) (70,) (90,)

add to accumulating sum

return the sum

helper function for sum()

call it ListAdd.

void ListAdd(L, P, S)

i^{th} row of A

i^{th} row of B

i^{th} row of Sum

ListAdd(L, P, S)

L : (10, ·) (30,) (50, x) (60,) (70,)

P : (20, ·) (40,) (50, y) (80,)

COPY COPY

S : (10, ·) (20, ·) ... (50, x+y)

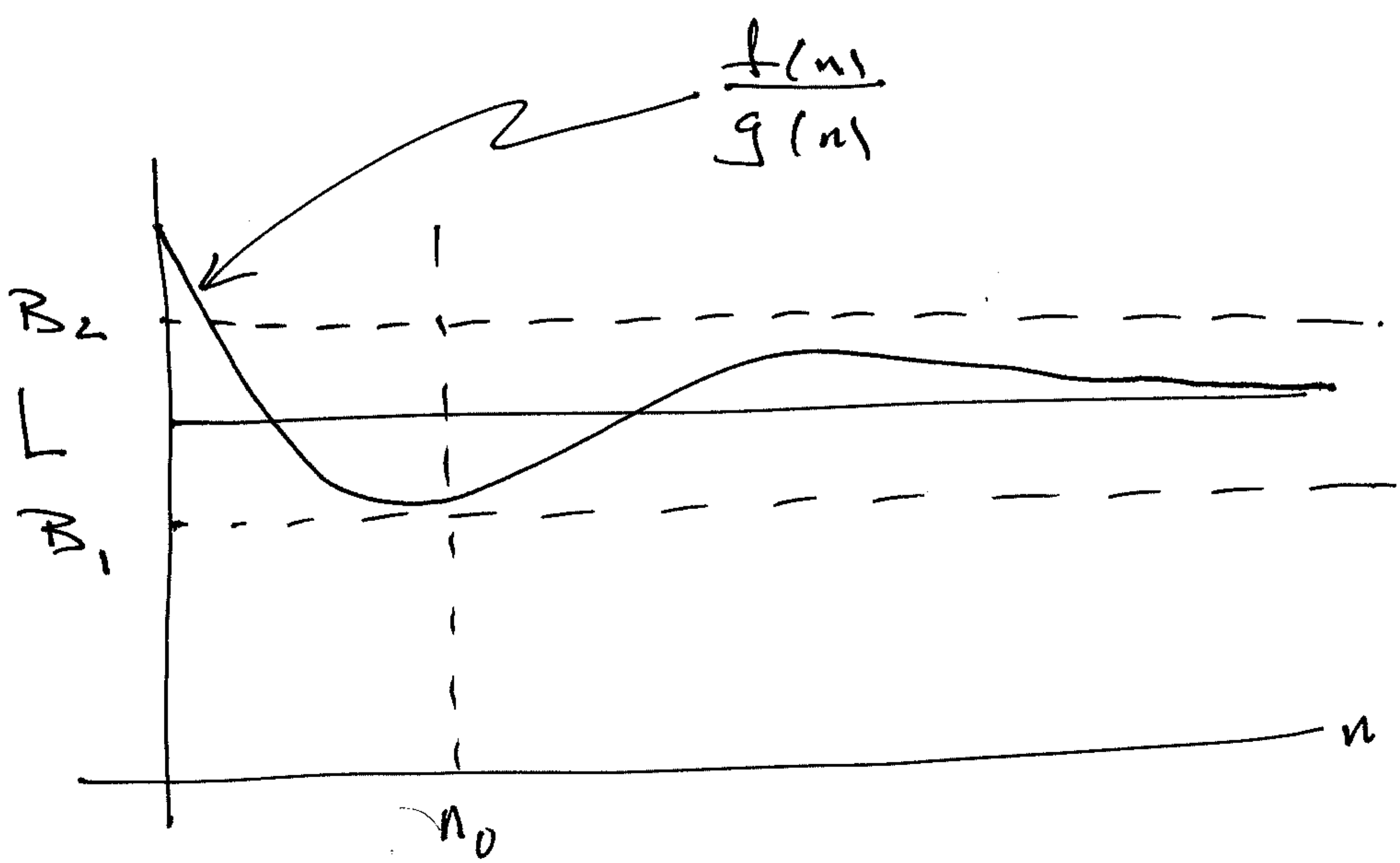
diff() is similar to sum().

Asymptotic growth of functions:

Thm

If $\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = L$ where $0 < L < \infty$,

then $f(n) = \Theta(g(n))$



note: converse is false.

Exercises:

(1) for any $\alpha, \beta \in \mathbb{R}$

$$n^\alpha = \begin{cases} o(n^\beta) & \text{if } \alpha < \beta \checkmark \\ \Theta(n^\beta) & \alpha = \beta \checkmark \\ \omega(n^\beta) & \alpha > \beta \checkmark \end{cases}$$

$$\frac{n^\alpha}{n^\beta} = n^{\alpha-\beta} \xrightarrow[\text{as } n\to\infty]{} \begin{cases} 0 & \text{if } \alpha < \beta \\ 1 & \alpha = \beta \\ \infty & \alpha > \beta \end{cases}$$

(2) for any $a, b \in \mathbb{R}$ with $a > 1$, $b > 1$:

$$a^n = \begin{cases} o(b^n) & \text{if } a < b \quad \checkmark \\ \Theta(b^n) & a = b \quad \checkmark \\ \omega(b^n) & a > b \quad \checkmark \end{cases}$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \xrightarrow[\text{as } n \to \infty]{} \begin{cases} 0 & \text{if } a < b \\ 1 & a = b \\ \infty & a > b \end{cases}$$

(1) compare 2^{3^n} to 3^{2^n}

$$\lim_{n \to \infty} \left(\frac{2^{3^n}}{3^{2^n}} \right) = \infty$$

$$\ln\left(\frac{2^{3^n}}{3^{2^n}}\right) = \ln\left(2^{3^n}\right) - \ln\left(3^{2^n}\right)$$

$$= 3^n \cdot \ln(2) - 2^n \cdot \ln(3) \to \infty$$

$\therefore\ 2^{3^n} = \omega\left(3^{2^n}\right)$