CSE 101-02 2-21-23

- Pa 5 : 1 last day ext.
- mid 2 : Thur. of next week.

Pa5:

client :

$$L = [\ 2 \quad 3 \mid 5\ ]$$

insertBefore(7)

$$L = [\ 2 \quad 3 \quad 7 \mid 5\ ]$$

A.concat(B)

inside

```
List L;
:
:
:
return L;
```

Pa 6 :

Sign - magnitude representation

int sign

magnitude List

Ex. addition $b = 100$, $p = 2$

```
    1     1     1
  ( 88    21    33)
  ( 65    91    79)
-------------------
  ( 01    54    13    12)
```

Another way

(88 21 33)

(65 91 79)

+1 +1 +1

(153 112 112) Vector sum

154 113

-100 -100 -100

+1 (01 54 13 12) normalize

→ return

Ex. Subtraction $b=100$, $p=2$

$$\begin{array}{ccc} {\scriptstyle -1} & {\scriptstyle -1} & \\ (88 & 21 & 33) = 882,133 \\ (65 & 91 & 79) = 659,179 \end{array}$$

$$(22 \quad 29 \quad 54) = 222,954$$

another way:

$$\begin{array}{ccc} (88 & 21 & 33) \\ (65 & 91 & 79) \end{array}$$

$$\begin{array}{ccc} {\scriptstyle -1} & {\scriptstyle -1} & \\ (23 & -70 & -46) \end{array}$$ vector diff.

$+100 \quad +100$

$\boxed{+1}$ $(22 \quad 29 \quad 54)$ normalize

↳ return

Ex. Subtraction $b=100$, $P=2$

$$\begin{array}{r} (65 \quad 91 \quad 79) \\ (88 \quad 21 \quad 33) \\ \hline \end{array}$$

$(-23 \quad 70 \quad 46)$

↑ leftmost digit negative

$$\boxed{-1} \quad \begin{array}{r} {\scriptstyle -1} \quad\quad {\scriptstyle -1} \\ (23 \quad -70 \quad -46) \\ \quad\quad -71 \\ \quad\quad 100 \quad\quad 100 \end{array}$$

$\boxed{-1}$ $(22 \quad 29 \quad 54)$ normalize

→ helper fcn. normalize returns a sign.

Note: what is the zero state

sign = 0

magnitude = [ ]

Ex. no-matize only. b=10

$$\begin{array}{cccccc} -9 & & +1 & +6 & -5 & -50 \\ (1 & & -90 & 9 & 73 & 0 & -500) \\ & & -89 & 15 & 68 & -50 \\ & & +90 & -10 & -60 & +50 & +500 \end{array}$$

$$(-8 \quad 1 \quad 5 \quad 8 \quad 0 \quad 0)$$

no-matize

$$\begin{array}{cc} & \begin{matrix} -1 & -1 & -1 & 0 & 0 & 0 \end{matrix} \\ \boxed{-1} & \begin{pmatrix} 8 & -1 & -5 & -8 & 0 & 0 \end{pmatrix} \\ & \quad\quad -2 \quad -6 \end{array}$$

$$\begin{array}{cc} & \quad\quad +10 \quad +10 \quad +10 \\ \boxed{-1} & \begin{pmatrix} 7 & 8 & 4 & 2 & 0 & 0 \end{pmatrix} \end{array}$$

check:

$$1 \cdot 10^5 + (-90) \cdot 10^4 + 9 \cdot 10^3 + 73 \cdot 10^2 + 0 \cdot 10 + (-500) \cdot 1$$

$$= -784{,}200$$

Ex. Multiplication b=10

```
    1 1
    1 2 3
  x 4 5 6
-----------
    7 3 8
  6 1 5 0
4 9 2 0 0
-----------
5 6 0 8 8
```

#digits = n

#digits = m

(1 2 3)

(4 5 6)

				Shift
(	6	12	18)	scalar mult 0
(	7	3	8)	normalize
(	5	10	15	0) scalar mult 1
(	5	17	18	8) add
(	6	8	8	8) normalize

(4 8 12 0 0) sc. mult.

(4 14 20 8 8) add

(5 6 0 8 8) normalize

Ex. mult b = 10

9 9 9

9 9 9

---

0 0 0

81 81 81 s.m.

81 81 81 add

(8 9 9 1) norm

81 81 81 0 s.m

89 90 90 1 add

(9 8 9 0 1) norm

(8 1 8 1 8 1 0 0) s.m

9 0 8 9 9 0 0 1 add

(9 9 8 0 0 1) norm

Exercise:

check that in base b:

$$(\underbrace{b-1, \ldots, b-1}_{n}, \ldots\ldots\ldots b-1)^2$$

$$= (\underbrace{b-1, \ldots, b-1}_{n-1}, b-2, \underbrace{0, \ldots, 0}_{n-1}, 1)$$

Ex. base 5: $(4444)_5^2$

$= (44430001)_5$

Runtime of mult.

$= \Theta(m \cdot n)$

more detailed analysis:

digit multiplications $= m \cdot n$

digit additions $= 2mn + m^2$