CSE 101-02 1-31-23

- Midterm1 : Thur 2/2/23
  - 1:30 - 2:35 exam
  - 2:35 - 2:40 break
  - 2:40 - 3:05 lecture

How to find SCCs of G:

- Run DFS(G), as vertices finish, Push them onto a stack.
- Compute Transpose G^T digraph

$V(G^T) = V(G)$

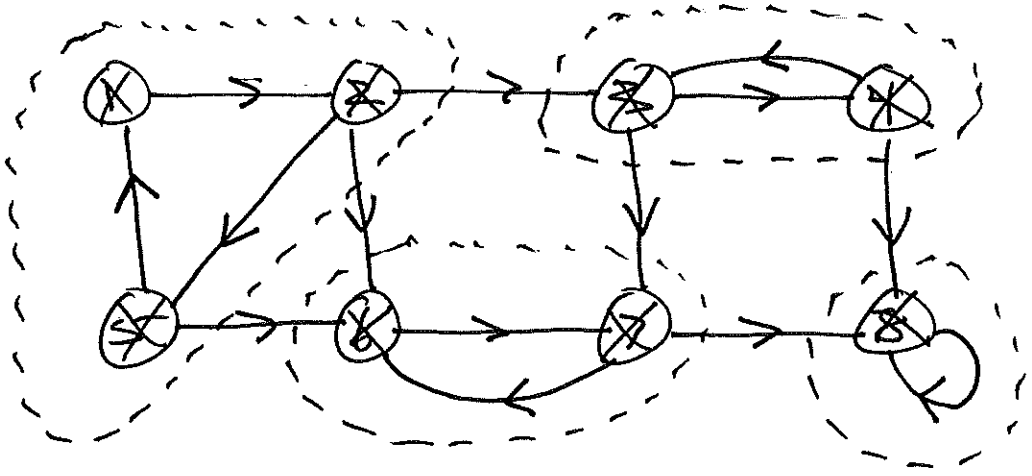
$E(G^T) = \{(y,x) \mid (x,y) \in E(G)\}$

- Run DFS(G^T), execute main loop by decreasing finish times from 1st call, i.e. stack order.

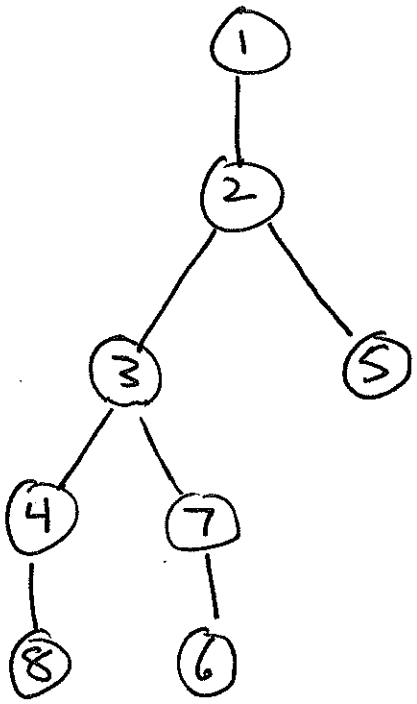
When done, the trees in the 2nd DFS forest constitute the SCCs of G.

Ex

G



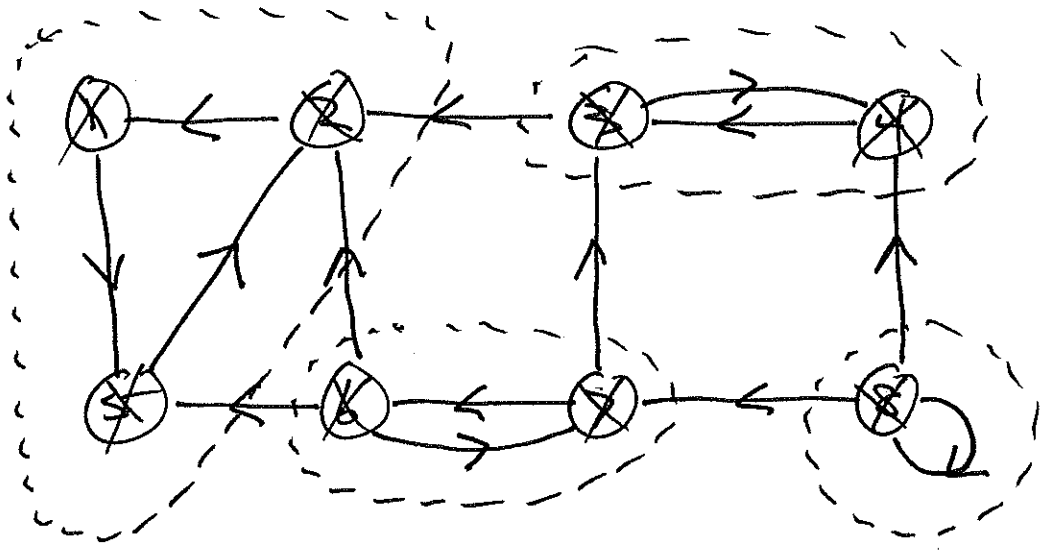
Forest



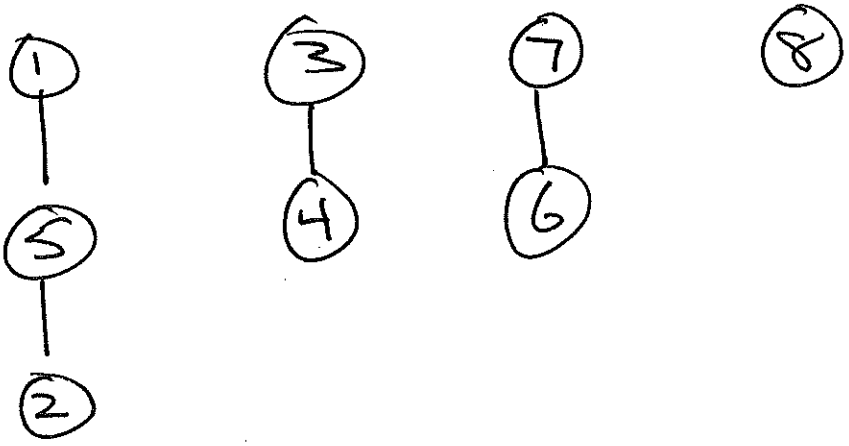
stack

1 ✓
2 ✓
5 ✓
3 ✓
7 ✓
6 ✓
4 ✓
8 ✓

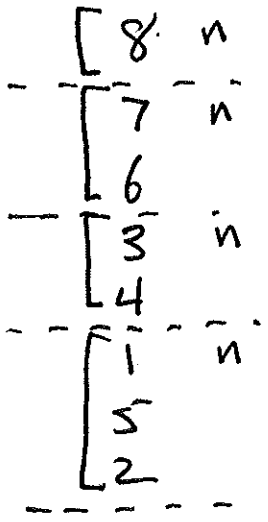
G^T



Forest



stack



components:

$C_1 = \{1, 2, 5\}$

$C_2 = \{3, 4\}$

$C_3 = \{7, 6\}$

$C_4 = \{8\}$

Defn

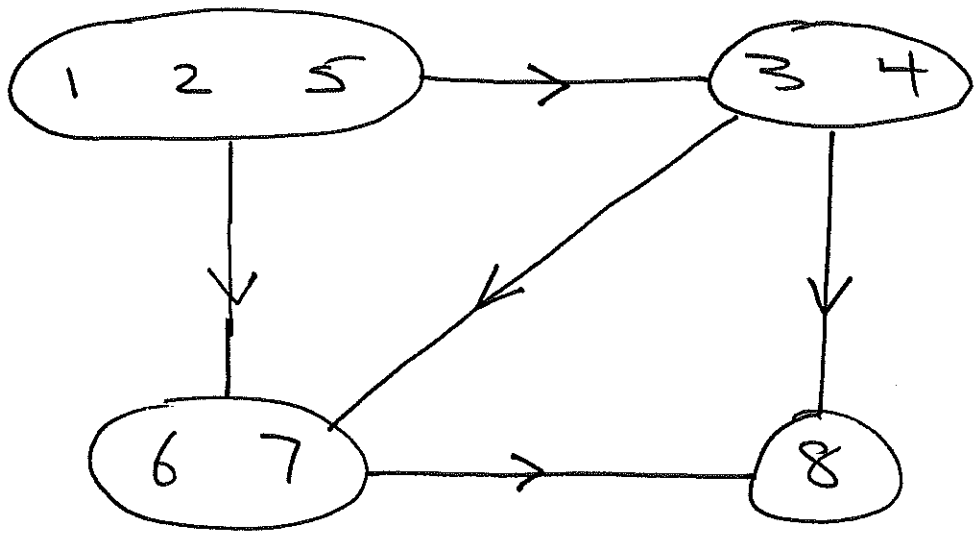
The component graph of a dig-raph G (also condensation graph) G^{scc} is the dig-raph with

$$V(G^{scc}) = \{ \text{scc}_i \text{ of } G \}$$

and

$$E(G^{scc}) = \{ (C_i, C_j) \mid \exists x \in C_i, \exists y \in C_j \text{ s.t. } (x,y) \in E(G) \text{ with } x \neq y \}$$

G^{scc} :



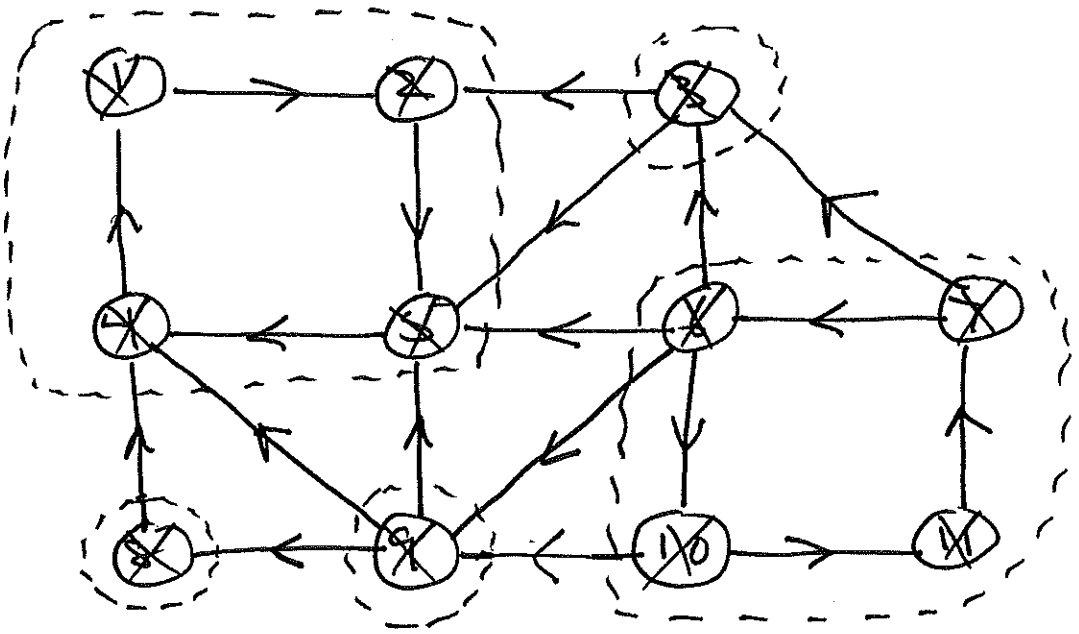
This is necessarily a DAG.

Pas output :

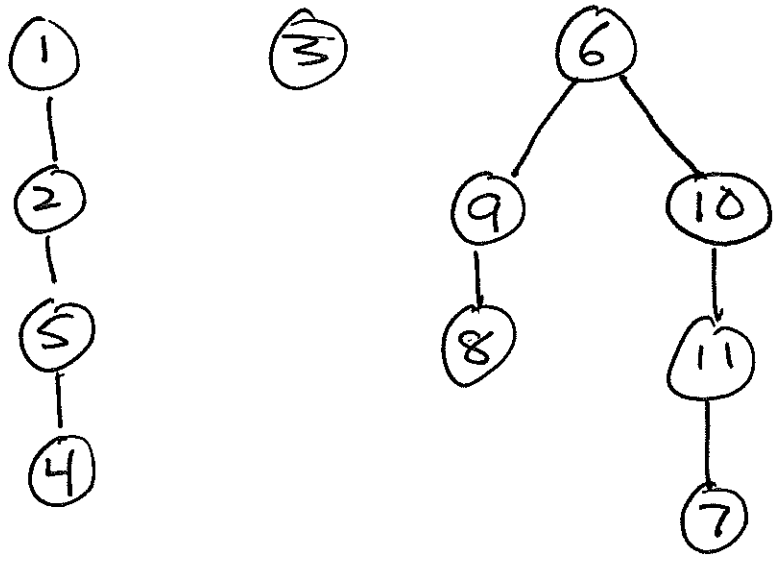
```
1 : 1 5 2
2 : 3 4
3 : 7 6
4 : 8
```

Ex.

G



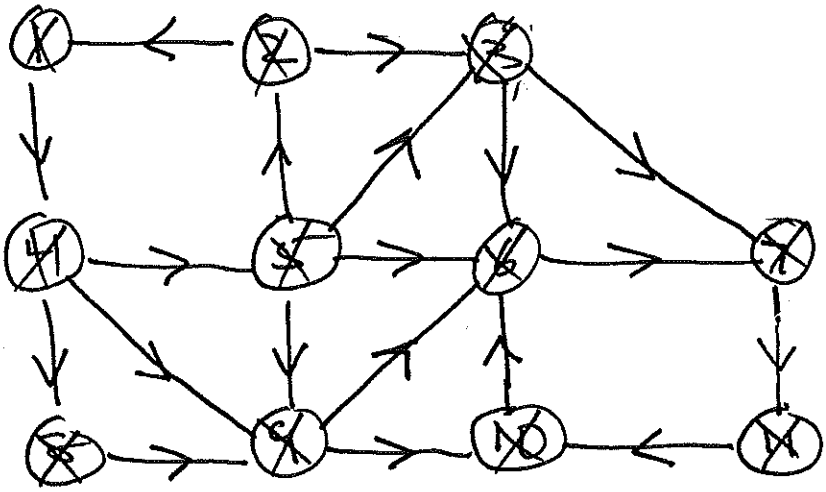
forest



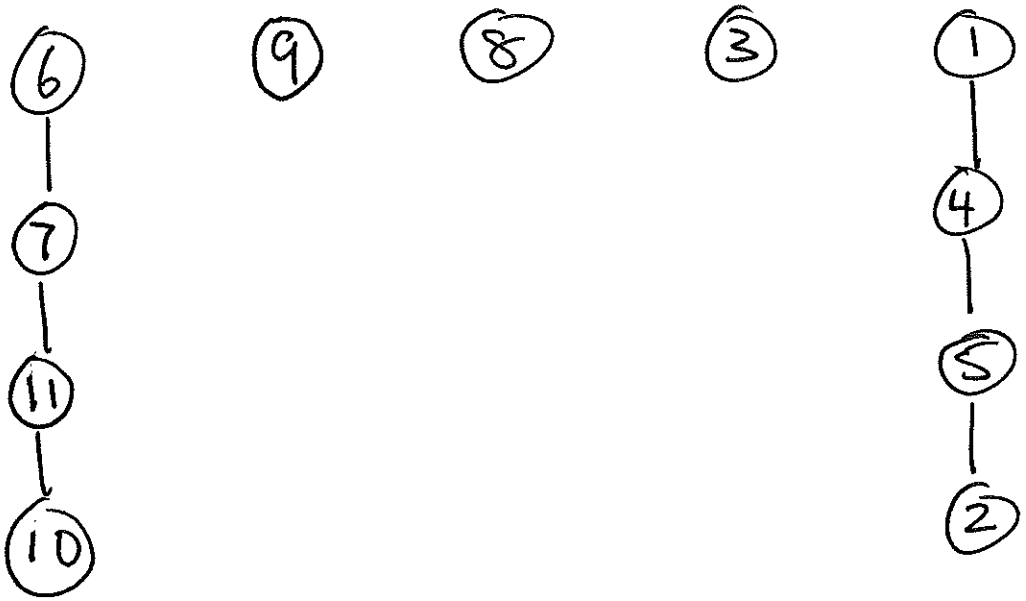
stack

6 ↙
10 ↙
11 ↙
7 ↙
9 ↙
8 ↙
3 ↙
1 ↙
2 –
5 –
4 –

G^T



forest



stack

1	n
4	
5	
2	
3	n
8	n
9	n
6	n
7	
11	
10	

Pa3 output:

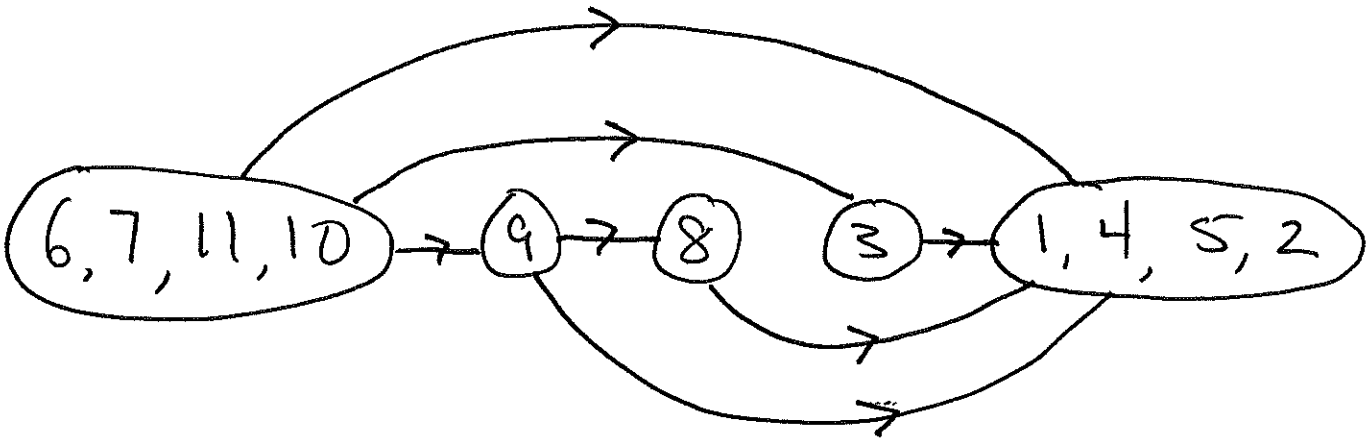
1: 6 7 11 10

2: 9

3: 8

4: 3

5: 1 4 5 2



Prototype for DFS in Pa3:

void DFS(Graph G, List S);

how to call it: