

Case 101-02

1-26-23

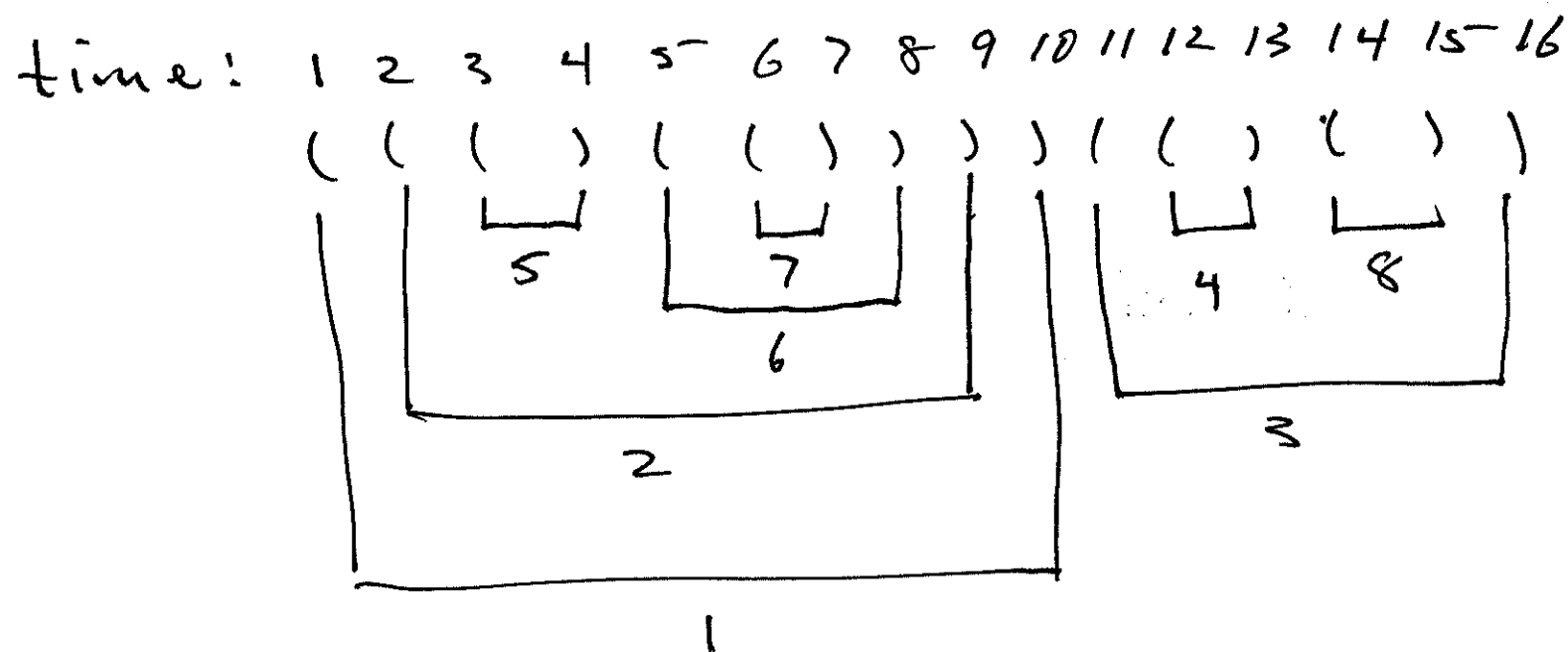
1

• Pa 2 : ext. 1 last day

• Pa 3 : posted

Ex (last time)

Parenthesis string!



Thm (white Path theorem)

let $x, y \in V(G)$. Then y is a descendant of x iff at time $d[x]$, G contains a white x - y Path.

Edge Classification

- Tree Edge: belong to DFS forest
- Back Edge: Joins a vertex to an ancestor
- Forward Edge: Joins a vertex to a descendant (not child)
- Cross Edge: all else
 - tree to tree
 - cousin to cousin

Ex. (last time)

Tree: (1,2), (2,5), (2,6), (6,7), (3,4), (3,8)

Back:

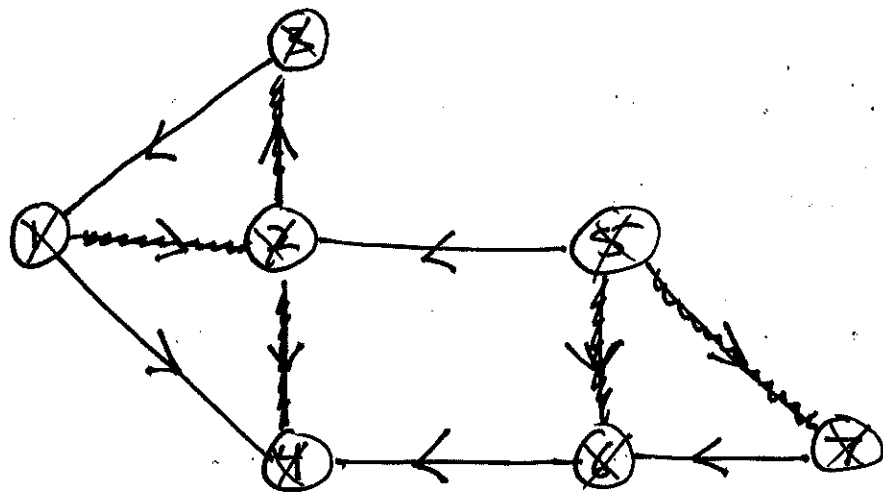
Forward: (1,5)

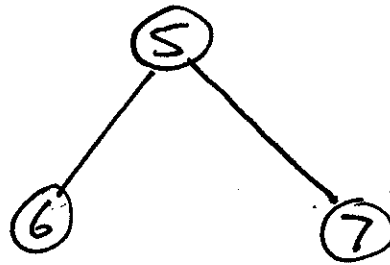
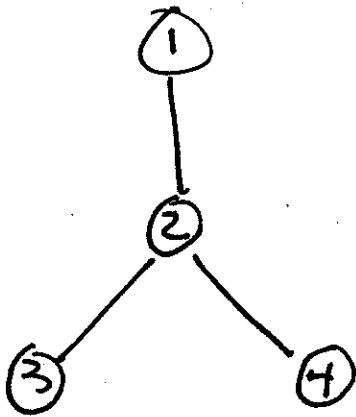
Cross: (3,2), (3,6), (3,7), (8,7), (6,5), (8,4)

tree-tree

cousin-cousin

EX.





Tree: $(1, 2), (2, 3), (2, 4), (5, 6), (5, 7)$

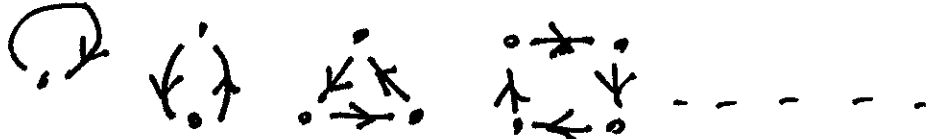
Back: $(3, 1)$

Forward: $(1, 4)$

Cross: $\underbrace{(5, 2), (6, 4)}_{\text{tree-tree}}, \underbrace{(7, 6)}_{\text{cousin-cousin}}$

Topological Sort

Restricted to digraphs

Cycle:  ...

Defn

$G = (V, E)$ is called acyclic iff it contains no cycles. Also:
DAG (directed acyclic graph)

Lemma

digraph G is acyclic iff $\text{DFS}(G)$ yields no back edges.

Equivalently :

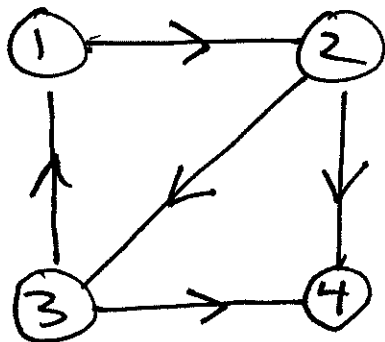
G contains a back edge iff it contains a directed cycle.

Defn

Let $G = (V, E)$ be a DAG. A

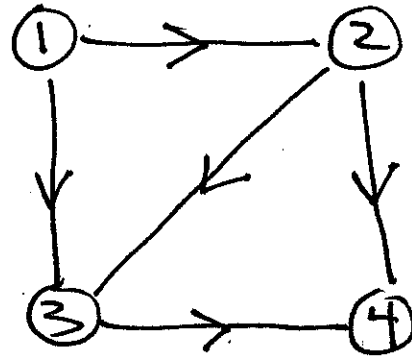
topological sort of G is an ordering of V such that if $(x, y) \in E$ then x is before y in the ordering.

Ex.



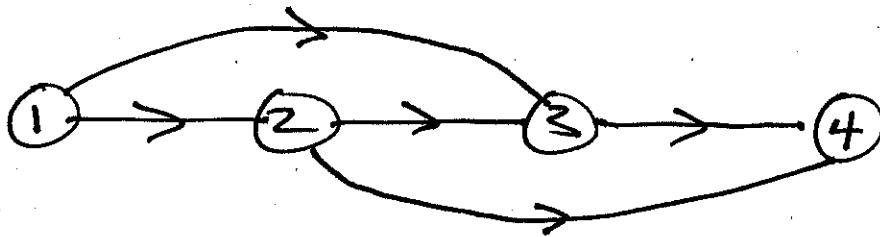
not
acyclic

Ex.



acyclic

Topological sort

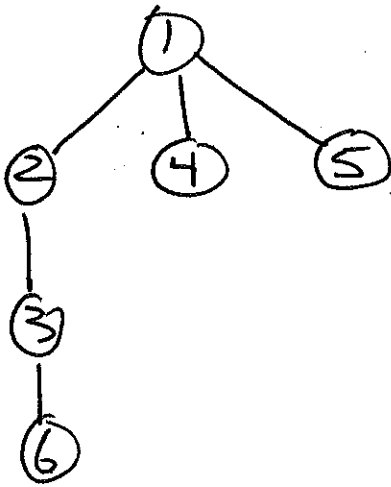
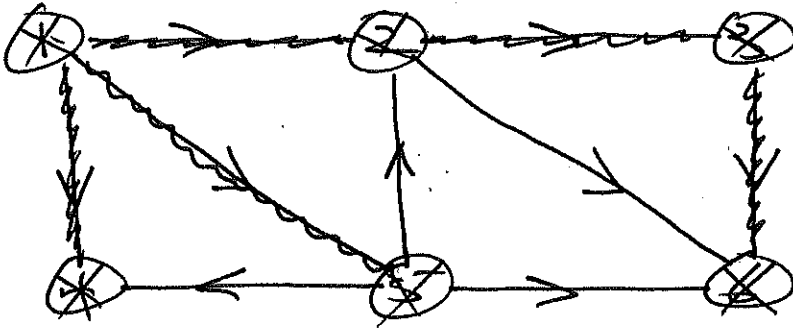


How to find a T.S. of a DAG.

- Run DFS(G)
- as vertices finish, push them onto a stack.

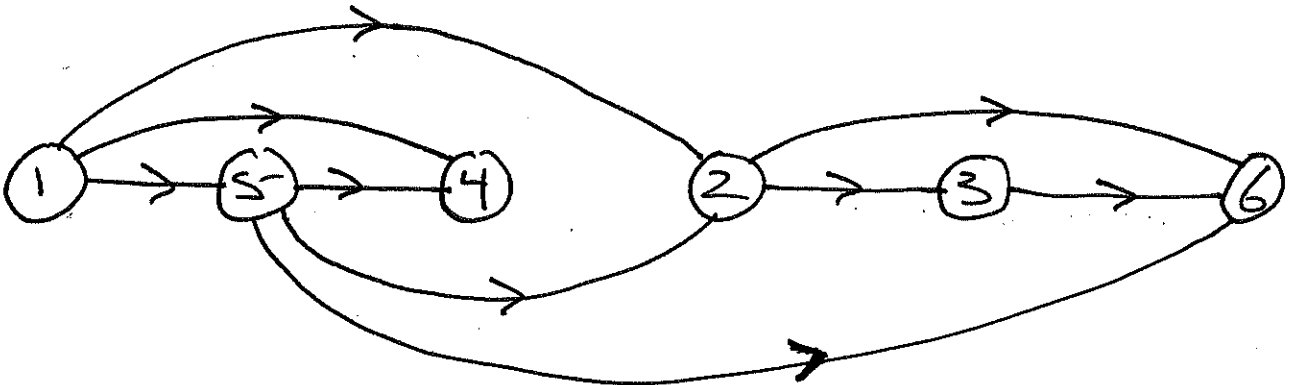
when done, stack is a T.S. of G .

Ex.



stack

1
5
4
2
3
6



Strongly connected components (SCC)

Let $G = (V, E)$ be a digraph. We say $y \in V$ is reachable from $x \in V$ iff G contains a (directed) x - y path. We say G is strongly connected iff for all $x, y \in V$: x is reachable from y , and y from x .

more generally: a subset $U \subseteq V$ is called strongly connected iff for all $x, y \in U$: x is reachable from y and y from x .

A subset $U \subseteq V$ is called a strongly connected component (SCC) of G iff both

- (1) U is strongly connected, and
- (2) U is maximal w.r.t. (2)

Ex.

