CSE 101-02 1-19-23

Def^n

A directed graph or dig-raph is

a pair of ~~sets~~

$$G = (V, E)$$

V: vertices, E: edges

where

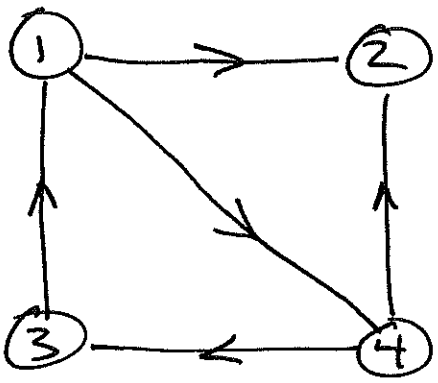
$V \neq \phi$ and

$E \subseteq V \times V = V^2$

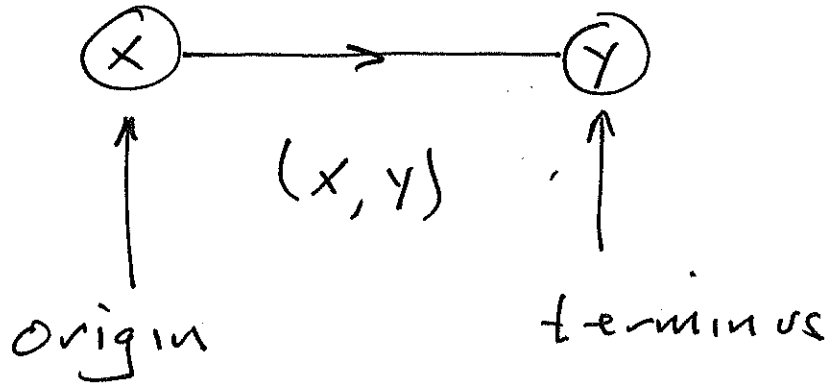
Ex.

$V = \{1, 2, 3, 4\}$

$E = \{(1,2), (1,4), (3,1), (4,2), (4,3)\}$



x	id(x)	od(x)
1	1	2
2	2	0
3	1	1
4	1	2



Representations of Graphs

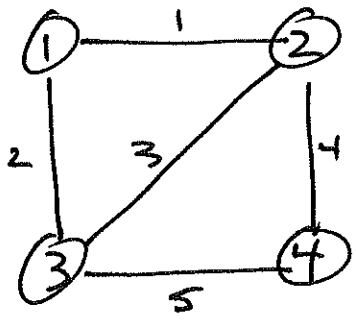
- Incidence matrix.

requires: $V = \{x_1, \ldots, x_n\}$

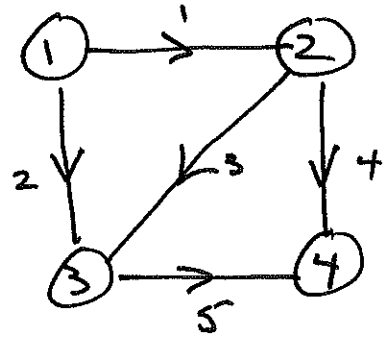
$E = \{e_1, \ldots, e_m\}$

$n \times m$ matrix

Ex.



G



D

$$\underline{\underline{I}}(G) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\underline{\underline{I}}(D) = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

- Adjacency matrix

  requires lables

  $V = \{x_1, x_2, \ldots, x_n\}$

  nxn matrix

$$A(G) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

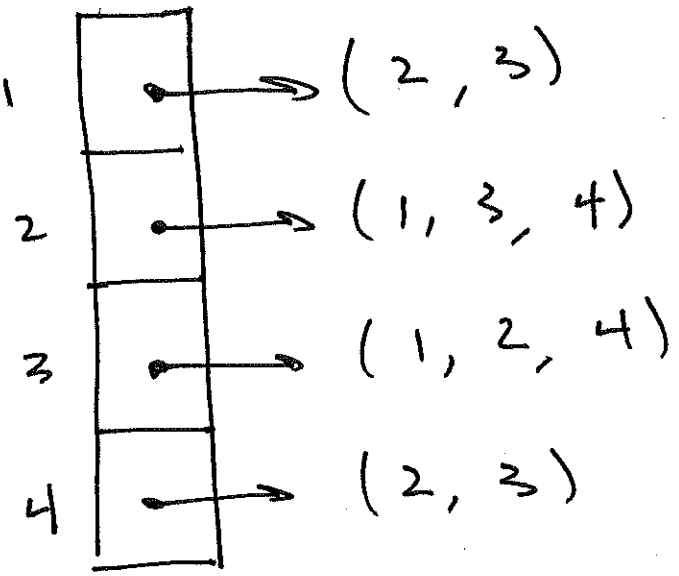
$$A(D) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Adjacency List

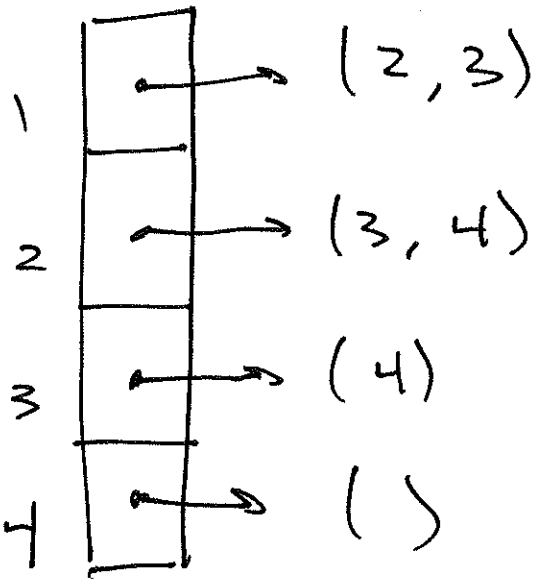
  require $V = \{x_1, \ldots, x_n\}$

  array of lists.

adj(G):



adj(D):



Handout: Graph Algorithms

Defn

Let $x, y \in V(G)$ (digraph or graph).

The distance from x to y is:

$$\delta(x,y) = \begin{cases} \text{min len. of an } x\text{-}y \text{ Path} \\ \text{if } y \text{ is reachable from } x. \\ \infty \quad \text{otherwise} \end{cases}$$

Single source shortest Paths (SSSP)

Given G and $s \in V(G)$ (called source) determine:

(1) $\delta(s,x)$ for all $x \in V(G)$.

(2) for each x with $\delta(s,x) < \infty$, a shortest s-x Path.

Breadth first search (BFS)

Vertex attributes: $x \in V(G)$

- color[x]: w undiscovered
  g discovered, not finished
  b finished
- distance[x]: $\delta(s, x)$
- Parent[x]: Predecessor of x along a shortest s - x Path.
- adj[x]

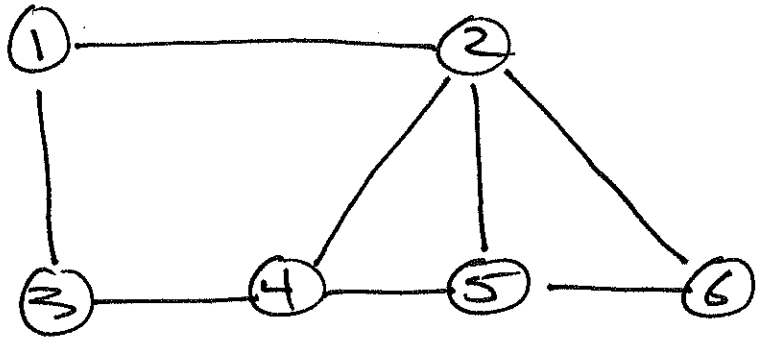
Need FIFO Queue: Q

Ex.

source

$s = 3$

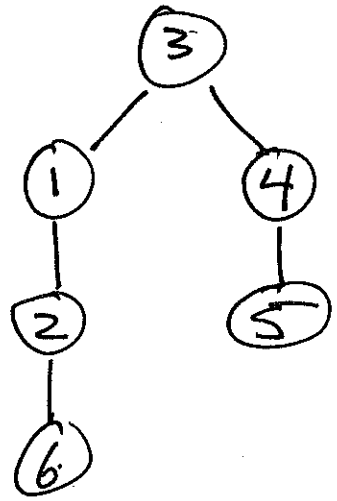
G



	adj	color	d	p
✗ = 1	2 3	~~w g~~ b	~~∞~~ 1	~~n~~ 3
✗ = 2	1 4 5 6	~~w g~~ b	~~∞~~ 2	~~n~~ 1
✗ = 3	1 4	~~g~~ b	0	n
✗ = 4	2 3 5	~~w g~~ b	~~∞~~ 1	~~n~~ 3
✗ = 5	2 4 6	~~w g~~ b	~~∞~~ 2	~~n~~ 4
✗ = 6	2 5	~~w g~~ b	~~∞~~ 3	~~n~~ 2

Q: ~~3~~ ~~1~~ ~~4~~ ~~2~~ ~~5~~ ~~6~~

BFS tree



dist

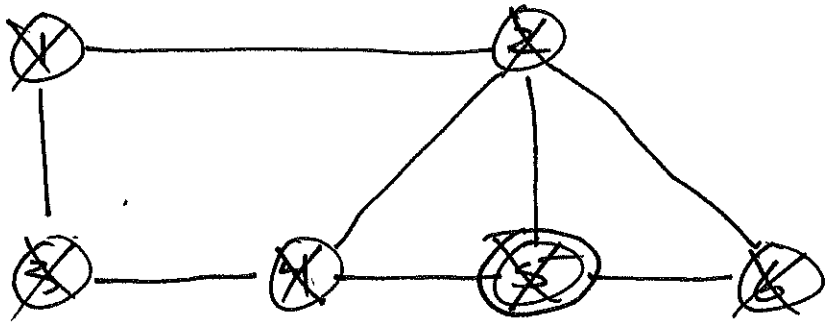
0

1

2

3

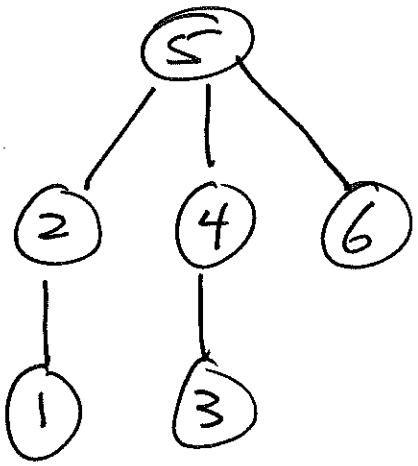
Ex.
Same graph, $s = 5$



Q: 5 2 4 6 1 3

dist

BFS Tree:

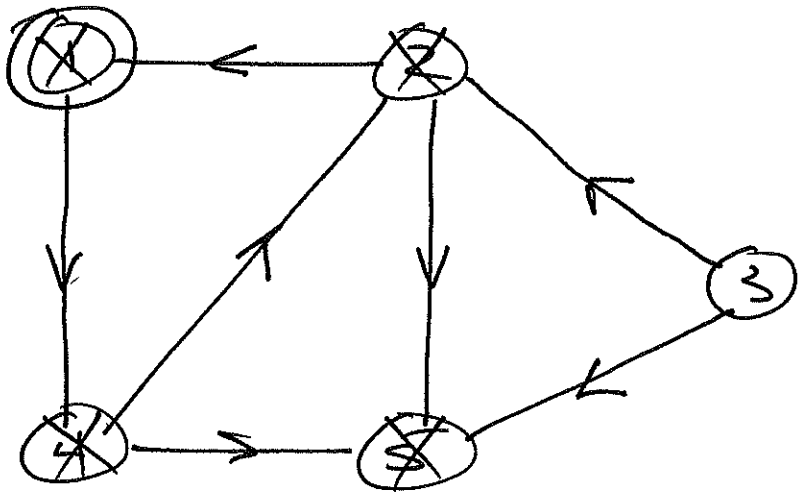


0

1

2

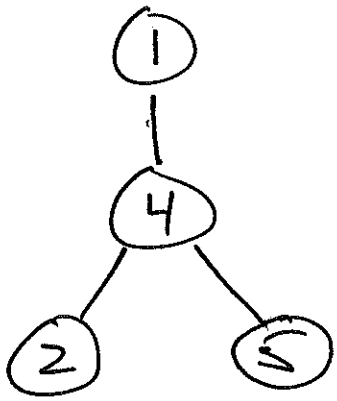
Ex. Source $\boxed{s = 1}$



Q: ~~1~~ ~~4~~ ~~2~~ ~~5~~

dist

BFS Tree



0

1

2