

CSE 101-01 3-16-23

• Pag: 1 last day ext. (Sat.)

continuing from p.m. lecture...

lemma

After $\text{Initialize}(G, s)$, the inequality

$$\delta(s, x) \leq d[x]$$

for all $x \in V(G)$ is maintained over any sequence of calls to $\text{Relax}(\cdot, \cdot)$,

Lemma 3

It

$$P: s = x_0, x_1, x_2, \dots, x_k = y$$

is a min-weight s - y path
in G , and the edges of P
are $\text{Relax}(\cdot, \cdot)$ 'ed in order

$$(x_0, x_1), (x_1, x_2), \dots, (x_{k-1}, x_k),$$

Then $d[y] = \delta[s, y]$. This is
true regardless of any other
relaxations that occur, even if
interspersed with the above
relaxations.

Runtime

$$n = |V|, m = |E|$$

• Initialize : $\Theta(n)$

• Relax($\cdot, \cdot$) : cost = $\Theta(1)$

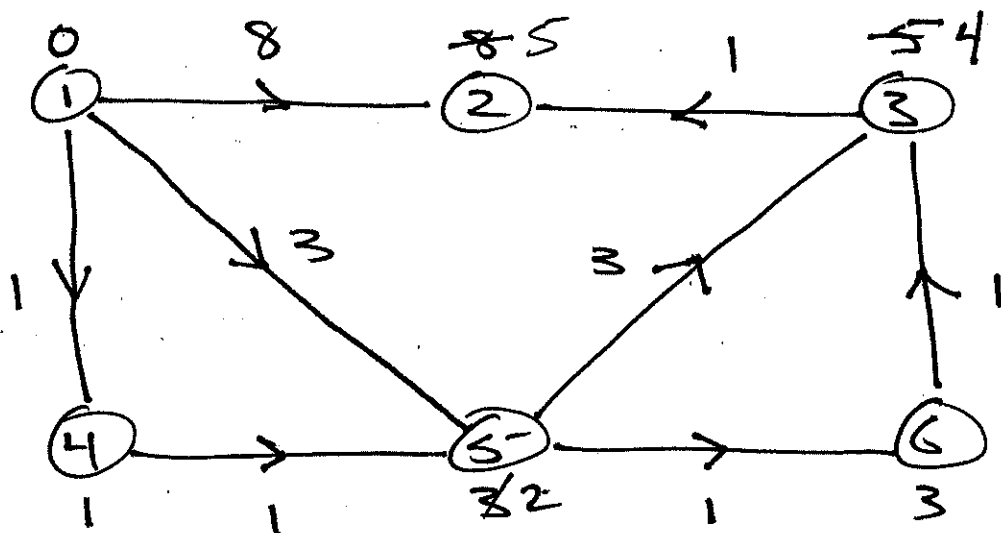
each edge is relaxed $n-1$ times, 1st loop costs:

$$\Theta(m(n-1)) = \Theta(mn)$$

• last loop costs : $\Theta(m)$

$$\begin{aligned} \text{Total cost} &= \Theta(n) + \Theta(mn) + \Theta(m) \\ &= \Theta(mn) \end{aligned}$$

Ex. $S = 1$



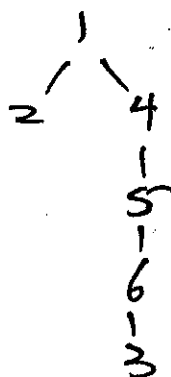
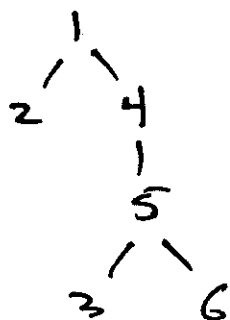
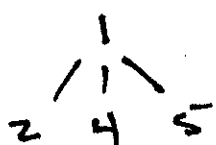
PQ : 1 2 3 4 5 6

d : 0 5 4 1 2 3

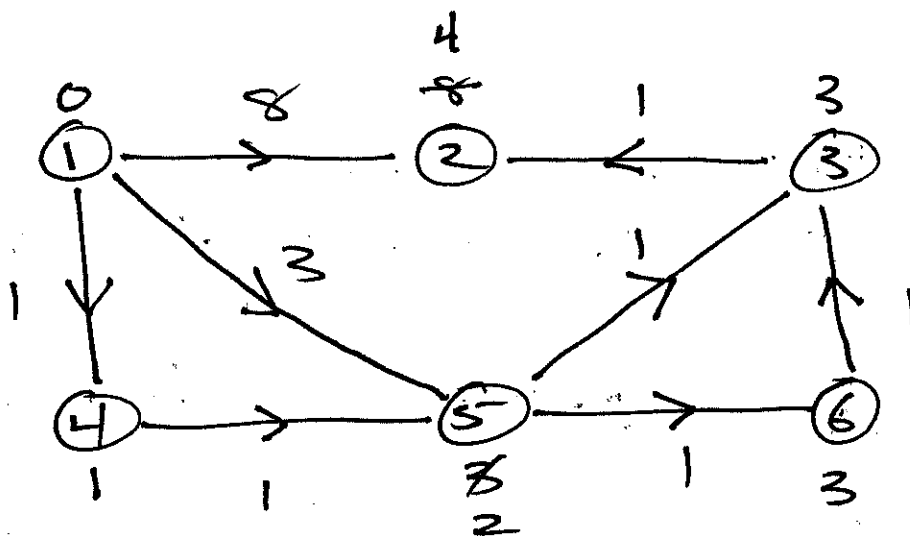
p : n 3 6 1 4 5

Remove order: 1 4 5 6 3 2

Tree:



Ex. $s = 1$



convention:

if 2 vertices have minimum d-value (keys), extract lowest label first.

PQ: 1 2 3 4 5 6

d: 0 4 3 1 2 3

P: 1 3 5 1 4 5

Remove order: 1 4 5 3 6 2

Tree!

