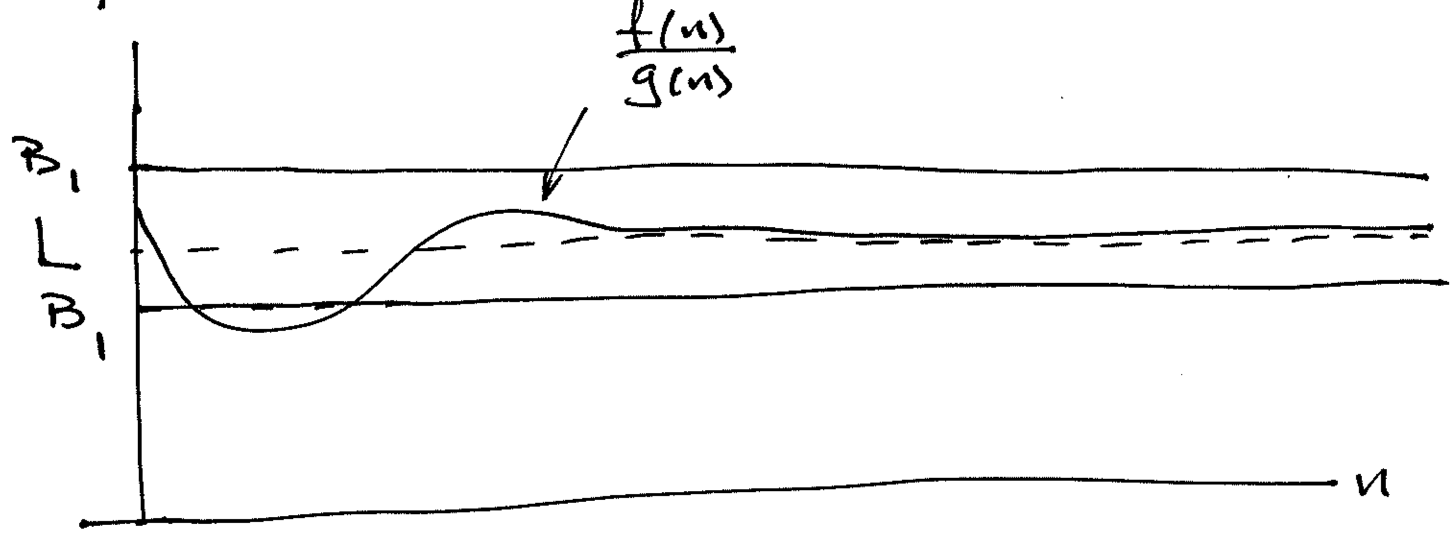
cse 101-01 2-9-23

Theorem

If $\lim_{n \to \infty} \left( \frac{f(n)}{g(n)} \right) = L$ and $0 < L < \infty$,

then $f(n) = \Theta(g(n))$.

why?



## Exercises:

(1) for any $\alpha, \beta$

$$n^\alpha = \begin{cases} o(n^\beta) & \text{if } \alpha < \beta \\ \Theta(n^\beta) & \alpha = \beta \\ \omega(n^\beta) & \alpha > \beta \end{cases}$$

$$\frac{n^\alpha}{n^\beta} = n^{\alpha-\beta} \longrightarrow \begin{cases} 0 & \text{if } \alpha < \beta \\ 1 & \alpha = \beta \\ \infty & \alpha > \beta \end{cases}$$

(2) for any $a, b \in \mathbb{R}$ with $a > 1, b > 1$:

$$a^n = \begin{cases} o(b^n) & \text{if } a < b \\ \Theta(b^n) & a = b \\ \omega(b^n) & a > b \end{cases}$$

why?

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \longrightarrow \begin{cases} 0 & \text{if} \quad a < b \\ 1 & \text{if} \quad a = b \\ \infty & \text{if} \quad a > b \end{cases}$$

(7) (last two fcns.)

compare $2^{3^n}$ to $3^{2^n}$

want: $\lim\limits_{n \to \infty} \left( \dfrac{2^{3^n}}{3^{2^n}} \right) = \infty$

$$\ln\left(\frac{2^{3^n}}{3^{2^n}}\right) = \ln\left(2^{3^n}\right) - \ln\left(3^{2^n}\right)$$

$$= 3^n \cdot \ln 2 - 2^n \cdot \ln 3 \longrightarrow \infty$$

$\therefore \quad 2^{3^n} = \omega\left(3^{2^n}\right)$

## SelectionSort (A)

```
n = length[A]
for i = 1 to n-1
        imin = i
        for j = i+1 to n
            if A[j] < A[imin]        ← Basic operation
                imin = j
        A[i] ↔ A[imin]      // swap
```

$$\underbrace{A_1 \cdots A_{i-1}}_{\text{sorted}} \; \underbrace{A_i \cdots\cdots A_n}_{\text{unsorted}}$$

(i-1) smallest

count # of array comparisons as a function of $n = length[A]$.

$$\#comp = (n-1) + (n-2) + (n-3) + \cdots + 1$$

Recall:

$$1 + 2 + \cdots + m = \frac{m(m+1)}{2}$$

$$\therefore \#comp = \frac{(n-1)(n-\cancel{1}+\cancel{1})}{2}$$

$$= \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$$

$$= \Theta(n^2)$$

## Runtime of BFS

$$n = |V(G)|, \quad m = |E(G)|$$

- initialize: $\Theta(n)$
- inner for-loop: $\Theta(m)$
- total cost $= \Theta(n) + \Theta(m)$
  $= \Theta(n+m)$.

## Runtime of DFS

- initialize : $\Theta(n)$
- main loop : $\Theta(n)$
- for loop in Visit( ) : $\Theta(m)$
- total cost $= \Theta(n+m)$

Pa4 stuff:

changeEntry(M, i, j, x)

① $x = 0$, $M_{ij} = 0$ : nothing

② $x = 0$, $M_{ij} \neq 0$ : delete

③ $x \neq 0$, $M_{ij} = 0$ : insert

④ $x \neq 0$, $M_{ij} \neq 0$ : overwrite

Product()

$$(A \cdot B)_{ij} = (\text{row } i \text{ of } A \cdot \text{col } j \text{ of } B)$$

$$= (\text{row } i \text{ of } A \cdot \text{row } j \text{ of } B^T)$$