

• Midterm 1 : Thur 2/2/23

- Start 9:50

end 10:55

break 10:55 - 11:00

lecture 11:00 - 11:25

• See review Problems.

• Pazz Due Friday

How to determine the SCCs
of a digraph?

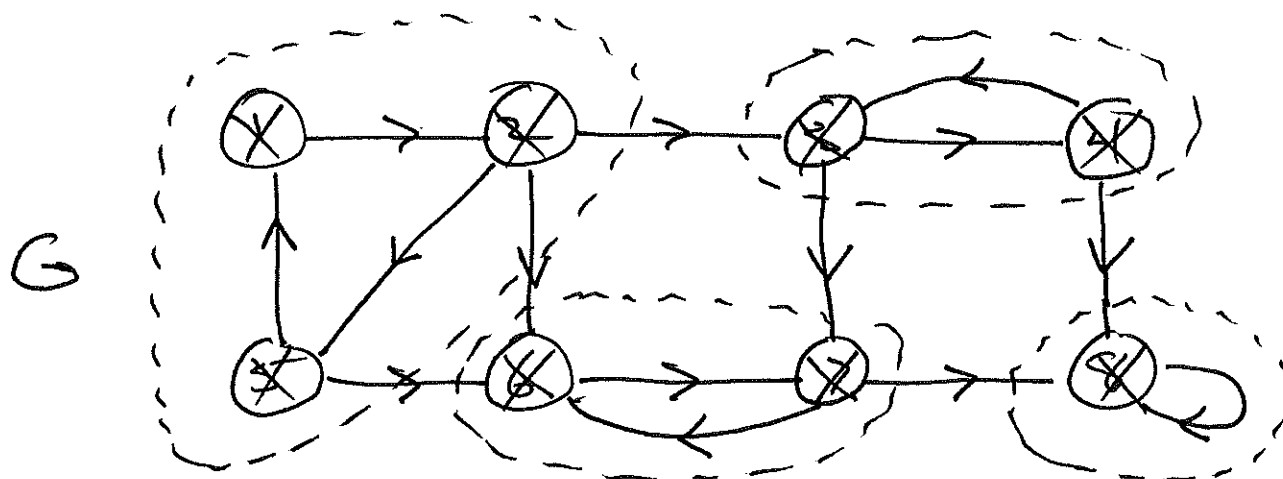
- Run $\text{DFS}(G)$, as vertices finish, push onto a stack.
- Compute Transpose G^T of G :

$$V(G^T) = V(G)$$

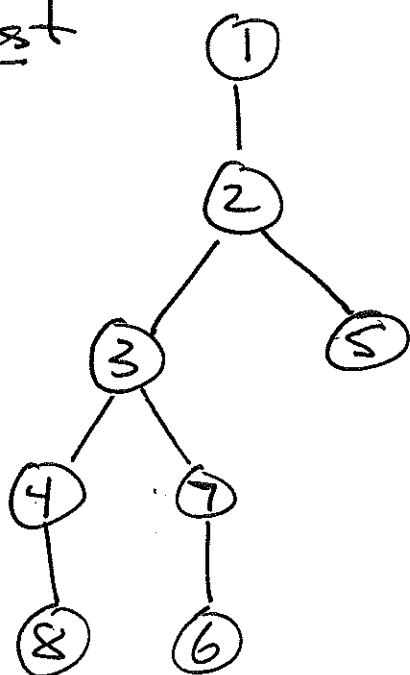
$$E(G^T) = \{(y, x) \mid (x, y) \in E(G)\}$$
- Run $\text{DFS}(G^T)$, executing main loop in decreasing finish time order from 1st call.

When done, the trees in DFS forest from 2nd call constitute the SCCs of G .

Ex.



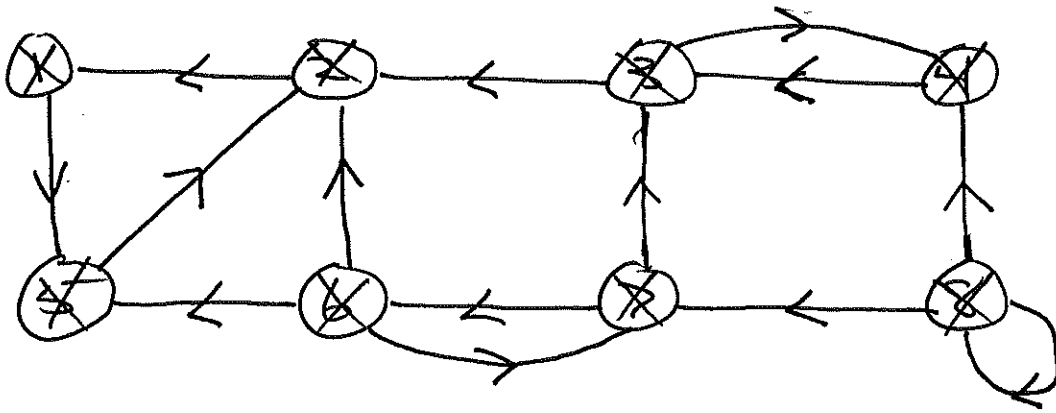
Forest



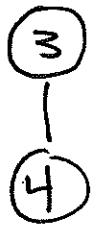
Stack :

1	✓
2	✓
5	✓
3	✓
7	✓
6	✓
4	✓
8	✓

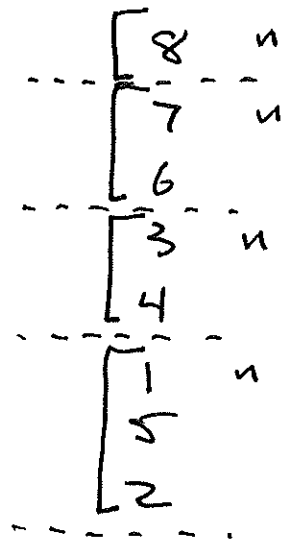
G



Forest



stack



SCCs

$C_1: \{1, 2, 5\}$

$C_2: \{3, 4\}$

$C_3: \{6, 7\}$

$C_4: \{8\}$

Topological
sorted
order

Defn

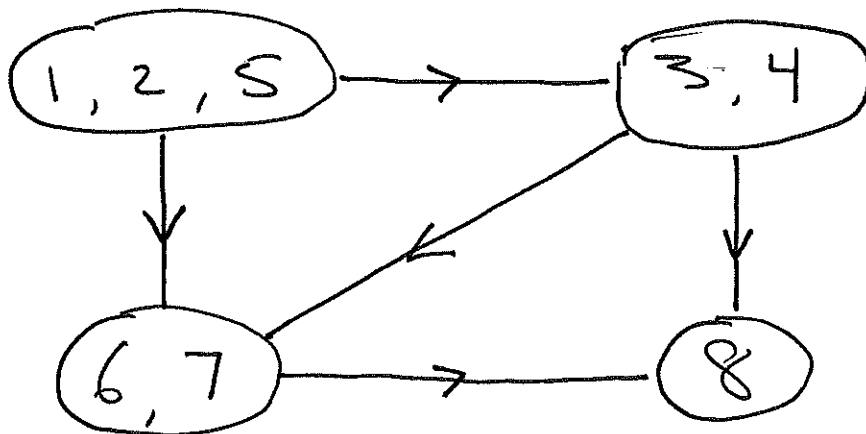
The component Graph or Condensation
graph of G is the Digraph
 G^{scc} where

$$V(G^{scc}) = \{ \text{SCCs of } G \}$$

$$E(G^{scc}) = \{ (C_i, C_j) \mid \exists x \in C_i, y \in C_j \text{ and } (x, y) \in E(G) \}$$

Ex.

G^{scc}



DAG

Par output :

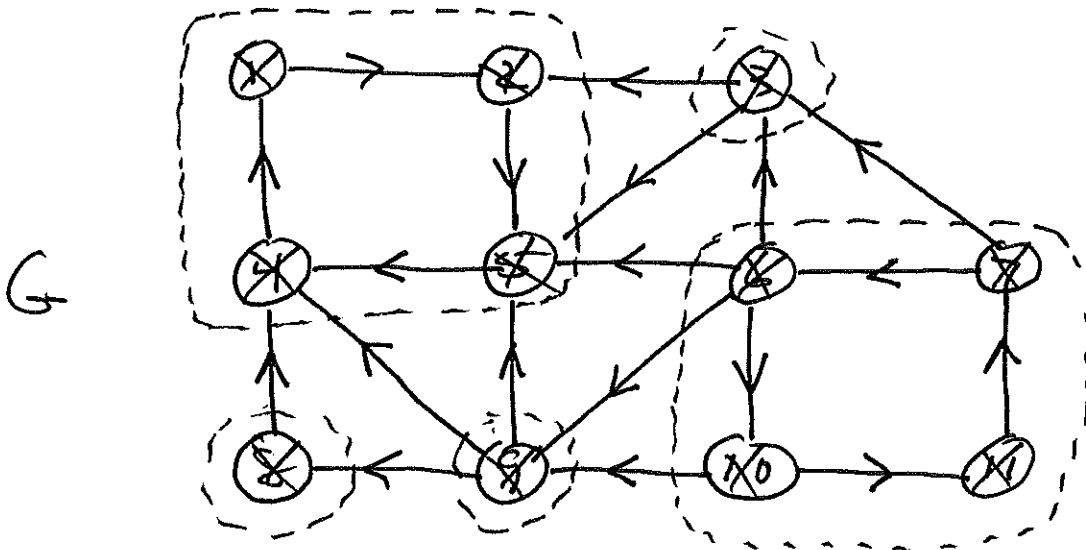
1 1 1 5 2

2! 3 4

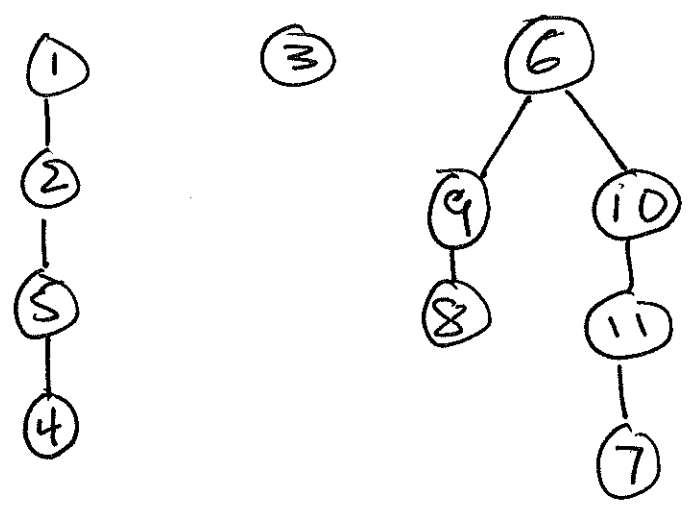
3! 7 6

4! 8

EX



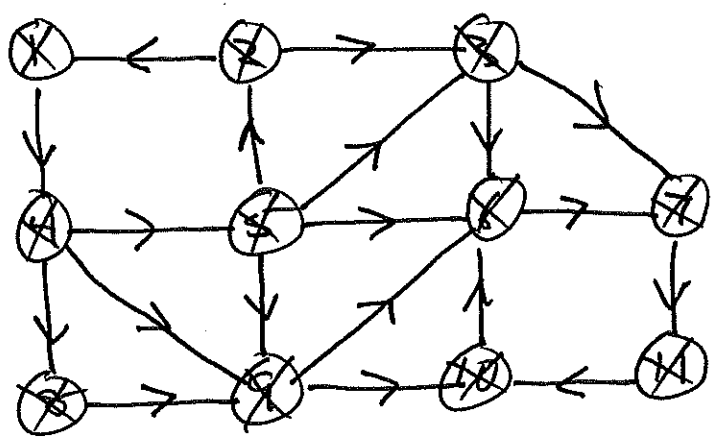
Forest



Stack

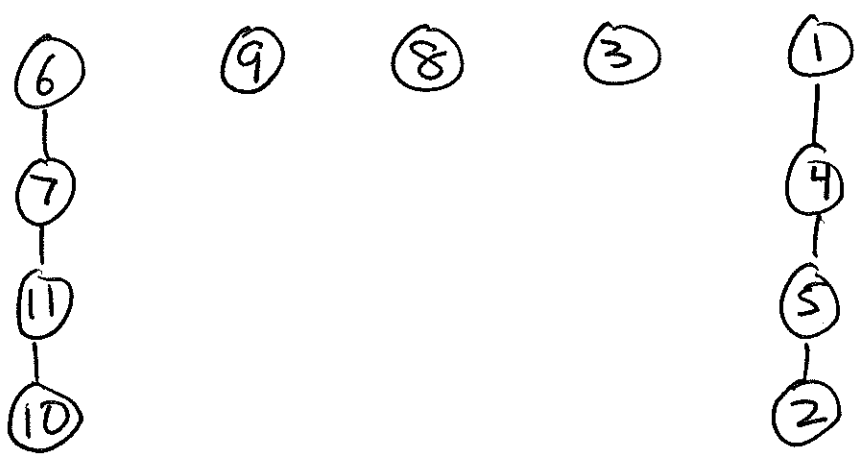
6 ✓
10 ✓
7 ✓
9 ✓
8 ✓
3 ✓
1
2
5
4

G_T



Stack

1 n
4 n
5 n
2 n
3 n
8 n
9 n
6 n
7
11
10



Par output file:

1: 6 7 11 10

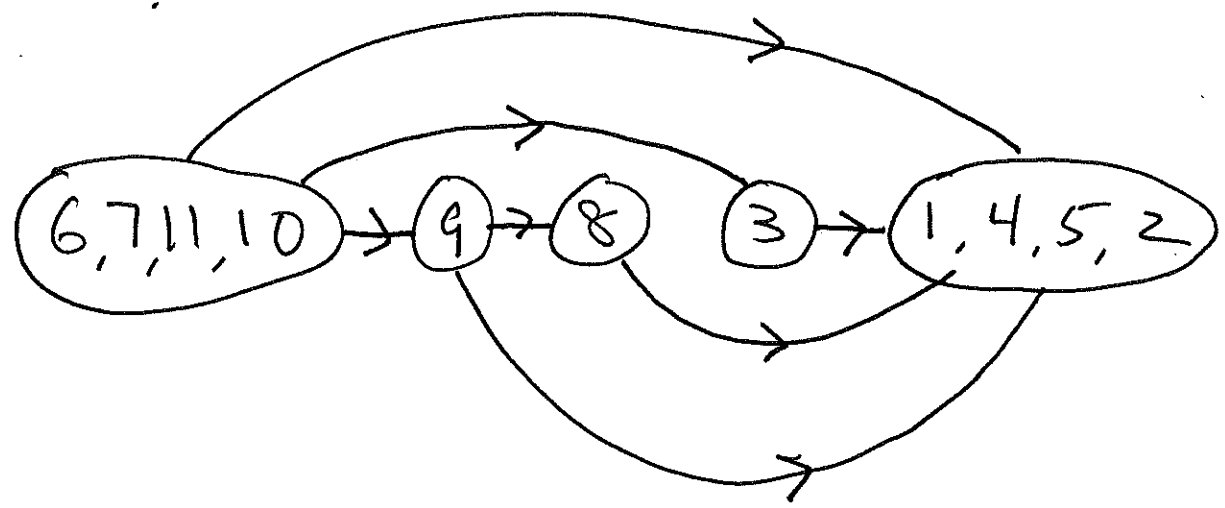
2: 9

3: 8

4: 3

5: 1 4 5 2

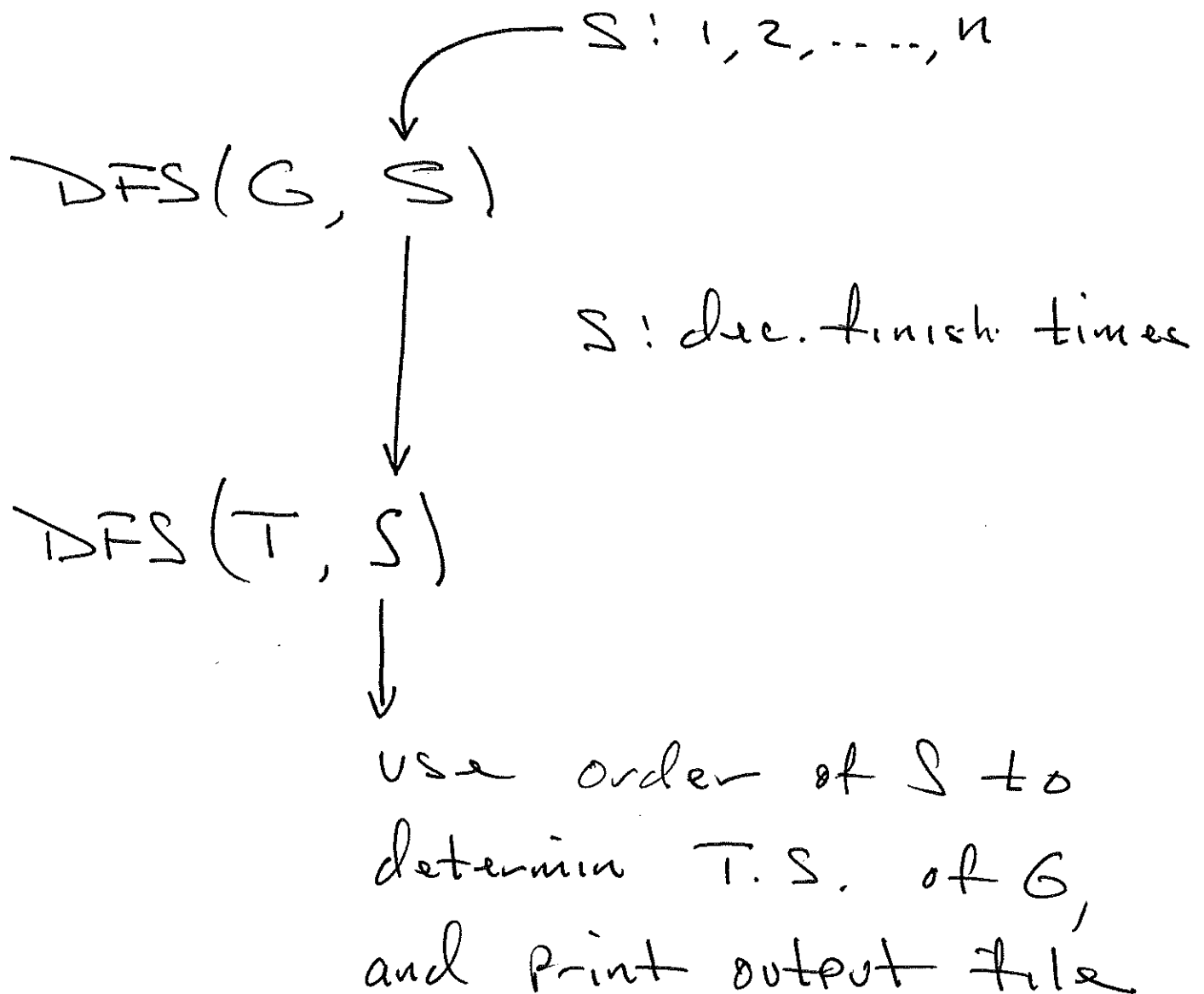
G^{sec}:



DFS Prototype (in PaaS)

void DFS(Graph G, List S);

call it like this:



Handout: Algorithm Runtime

- choose a basic operation and count the number of executions, as a function of the input size.

EX

OP1

for $i = 1$ to 10

OP2

for $j = 1$ to 10

OP3

for $k = 1$ to 10

OP4

OP	# executions
1	1
2	10
3	100
4	1000

total = 1111