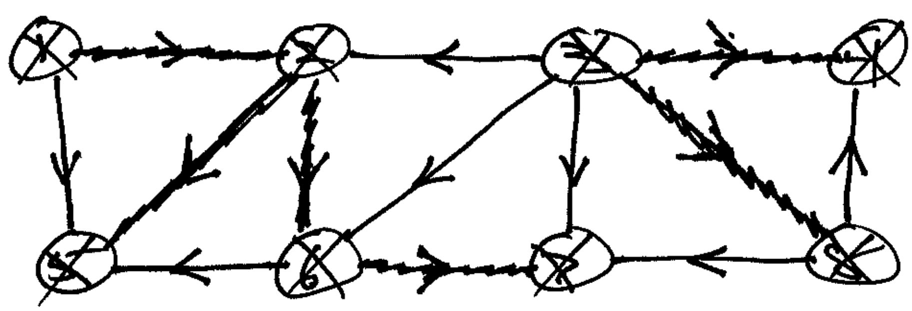
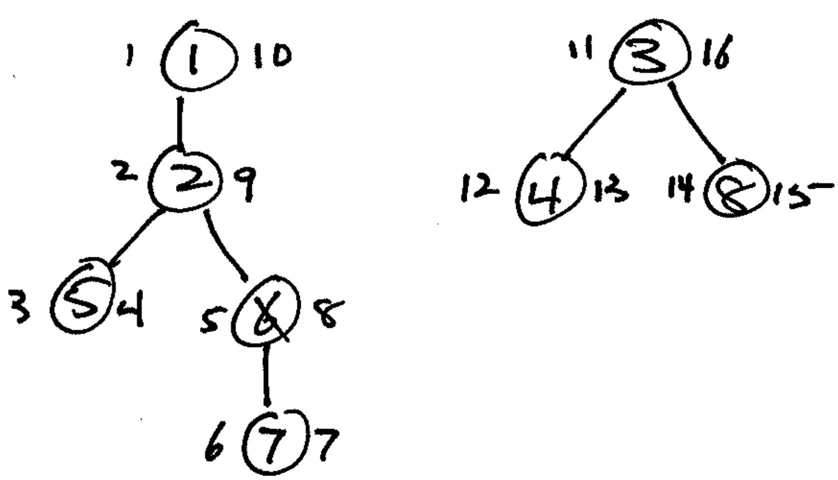
CSE 101 1-26-23

- PA3 : will post today
- PA2 : ext. 1 more day

Ex. (previous)





Parenthesis string:

time: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

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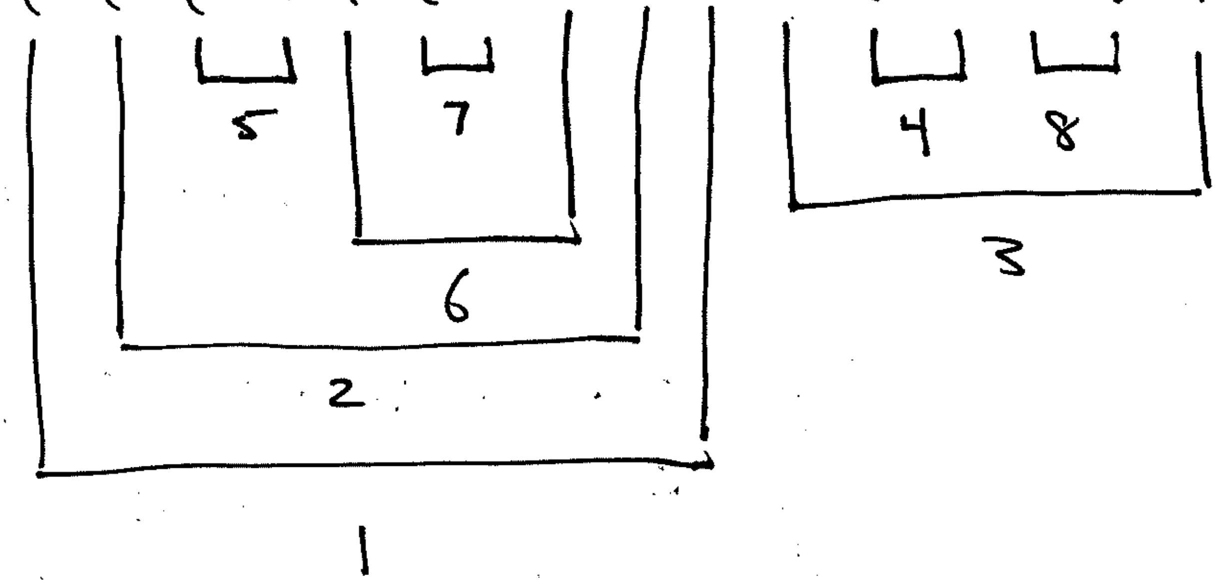


<u>Thm</u> (White Path theorem)

let $x, y \in V(G)$. Then y is a descendant of x iff at time $d[x]$, G contains an x-y path consisting entirely of white vertices.

Edge classification

- Tree edges: Belong to DFS Forest
- Back edges: Join a vertex to an ancestor
- Forward edges: join vertex to a descendant (other than child)
- cross edges: all other
  - tree to tree
  - cousin to cousin

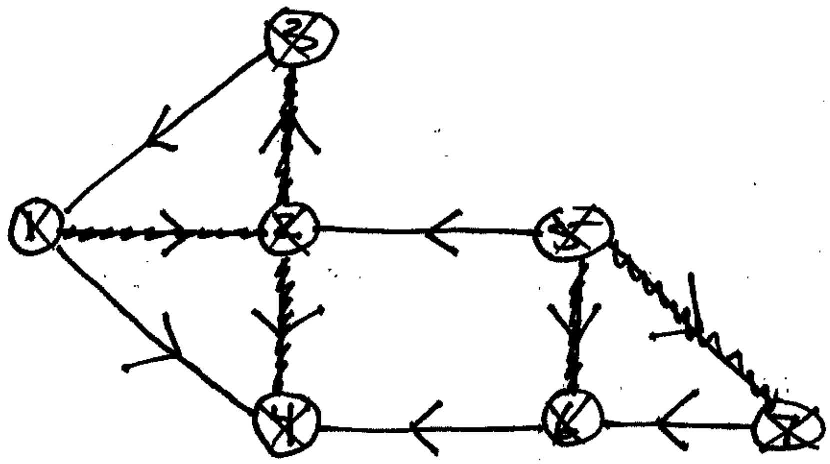
Ex. Tree: (1,2), (2,5), (2,6), (6,7), (3,4), (3,8)

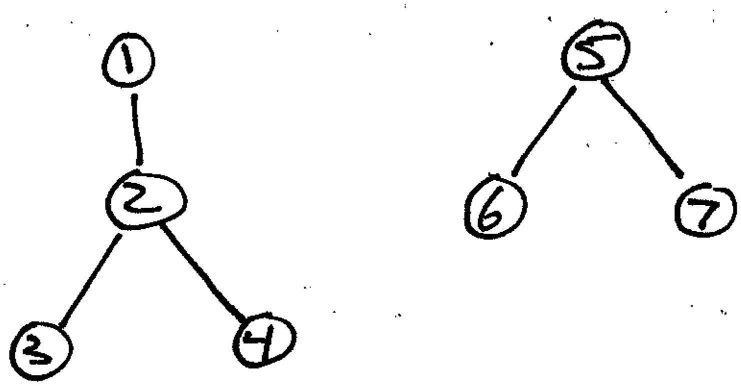
Back:

Forward: (1,5)

Cross: $\underbrace{(3,2),(3,6),(3,7),(8,7)}_{\text{tree-tree}}$, $\underbrace{(6,5),(8,4)}_{\text{cousin-cousin}}$

Ex.





Tree: (1, 2), (2, 3), (2, 4), (5, 6), (5, 7)

Back: (3, 1)

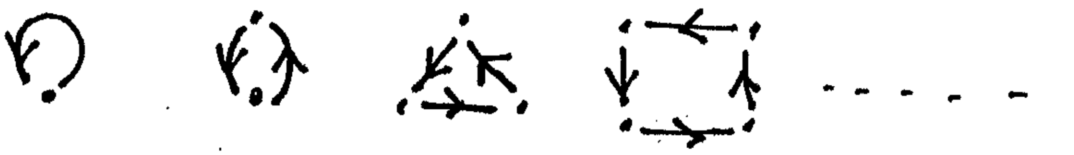
Forward: (1, 4)

Cross: (5, 2), (6, 4), (7, 6)

Topological Sort

Restrict to digraphs.

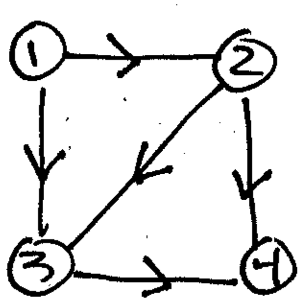
Directed cycle:



Defn

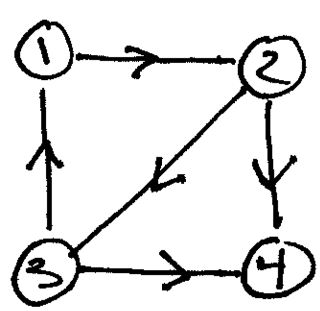
A digraph is *acyclic* iff it contains no cycles. called: DAG.

Ex.



acyclic
DAG

Ex.



not acyclic

Lemma

a digraph G is acyclic iff DFS(G) yields no back edges.

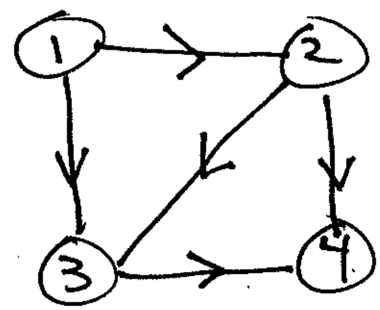
equivalently:

G contains a back edge iff it contains a directed cycle.

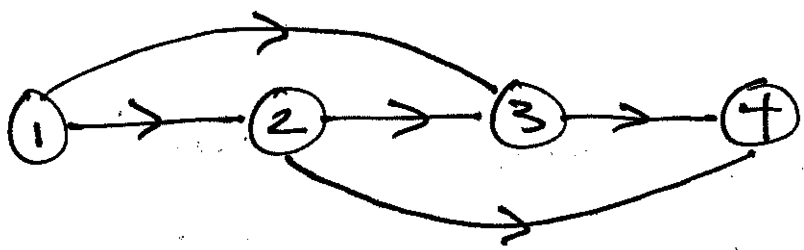
Defn

let $G = (V, E)$ be a DAG. A topological sort of G is an ordering of V such that if $(x, y) \in E$, then x is before y in the ordering.

Ex.



Topological sort:

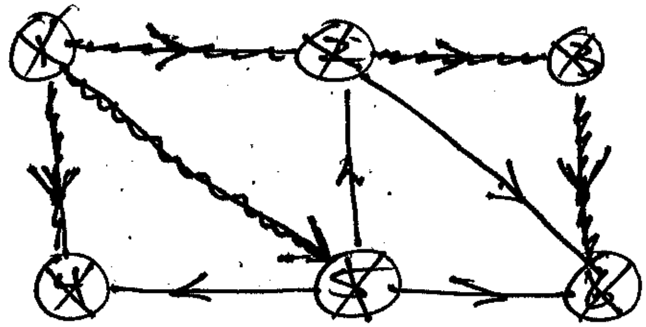


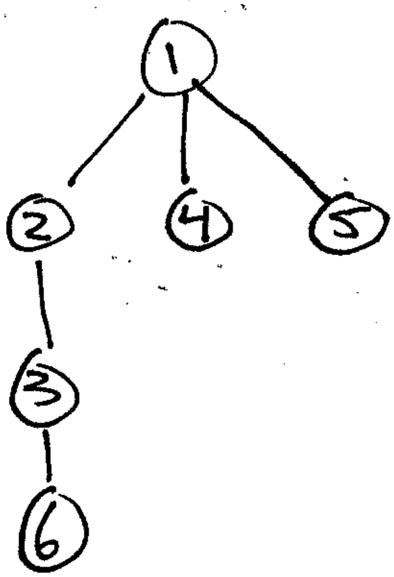
To find a T.S. in a DAG G:

- run DFS(G)
- as vertices finish, push them onto a stack.

When Done, the stack is a T.S. of G.

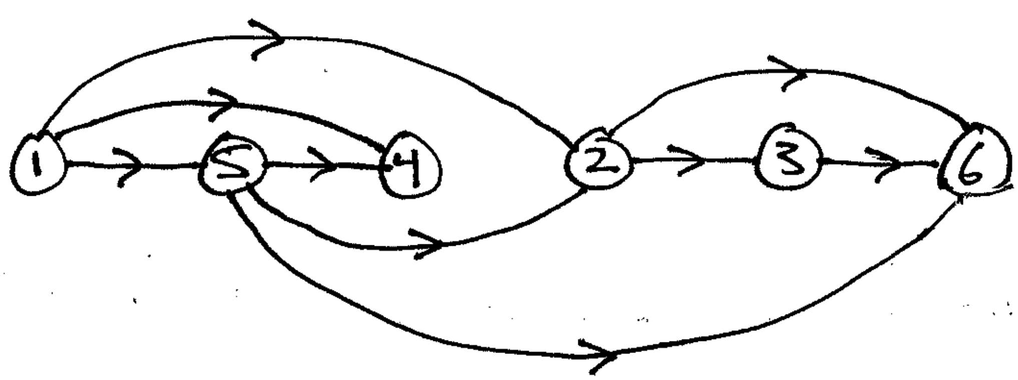
Ex.





stack

1
5
4
2
3
6



Exercise this DAG has 3 more topological sorts. find them.

Strongly connected components (SCC)

Let $G=(V,E)$ be a digraph.

y is *reachable* from x iff G contains a directed $x-y$ path

We say G is *strongly connected* iff for every $x, y \in V$, both x is reachable from y and y from x. Similarly a subset $U \subseteq V$ is *strongly connected* iff for all $x, y \in U$: x is reachable from y and y from x.

Defn

$U \subseteq V$ is said to be a **strongly connected component** (SCC) of G iff

(1) U is strongly connected

and

(2) U is maximal w.r.t. (1).

Ex.