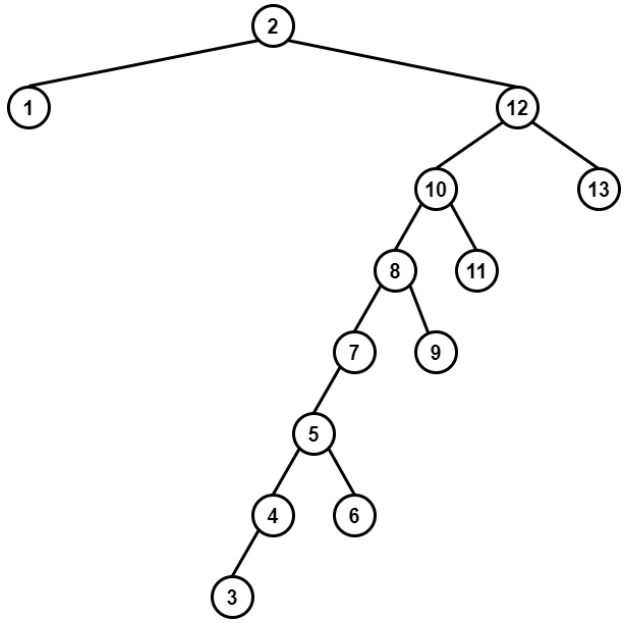
CSE 101
Winter 2022
Quiz 5 **Solutions**

1. (25 Points) Consider the Binary Search Tree pictured below.



Use the Theorem discussed in class (see notes from 3-1-22 page 1) to explain why it is **impossible** to assign colors Red and Black to the nodes in this tree so as to satisfy the RBT properties. Note: be sure to include the nil leaves (not pictured here) when calculating the height of this tree.

Solution:
According to the stated theorem, we must have $h \leq 2\lg(n+1)$ for any RBT with n internal nodes and (absolute) height h. Observe that the tree above has $n = 13$ and $h = 8$ (counting the nil children of node 3 as the deepest leaves.) But then

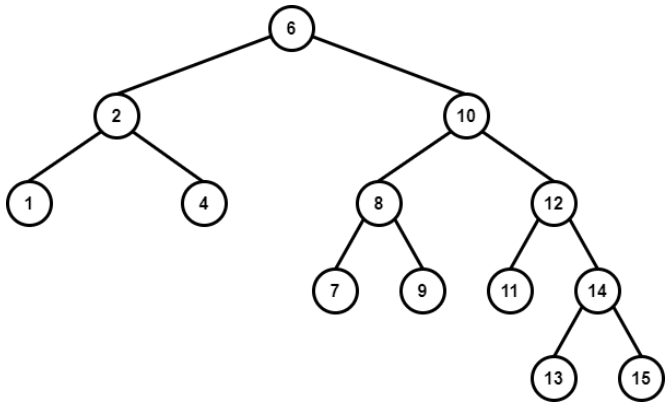
$$2\lg(n+1) = 2\lg(14) < 2\lg(16) = 2 \cdot 4 = 8 = h$$

making the inequality in the theorem is false. Therefore above tree cannot be colored so as to satisfy the RBT properties. ■

2. (25 Points) Use the BST TreeInsert() algorithm to insert the following keys: 6, 2, 1, 4, 10, 8, 7, 9, 12, 11, 14, 13, 15 (in order) into an initially empty BST.

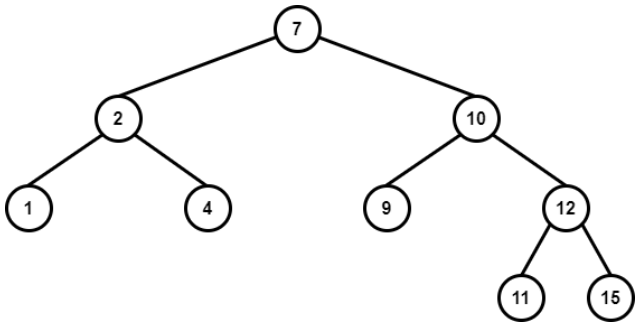
   a. (10 Points) Draw the resulting BST

   Solution:



   b. (10 Points) Use the BST Delete() algorithm to delete the following keys: 8, 6, 13, 14 (in order) from the tree you drew in part (a), and draw the resulting BST.

   Solution:



   c. (5 Points) Assign colors Red/Black to the nodes in the tree you drew in part (b) so as to satisfy the Red-Black tree properties, and so that $\mathrm{bh}(7) = 3$ and $\mathrm{bh}(12) = 1$. Note: just assign colors, do not run any RBT algorithm. Be sure to count the nil children when computing black-height. **State your answer by giving the set of keys to be colored Red.**

   Solution: $\mathrm{Red\ Nodes} = \{11, 15\}$