cse 101 2-17-22

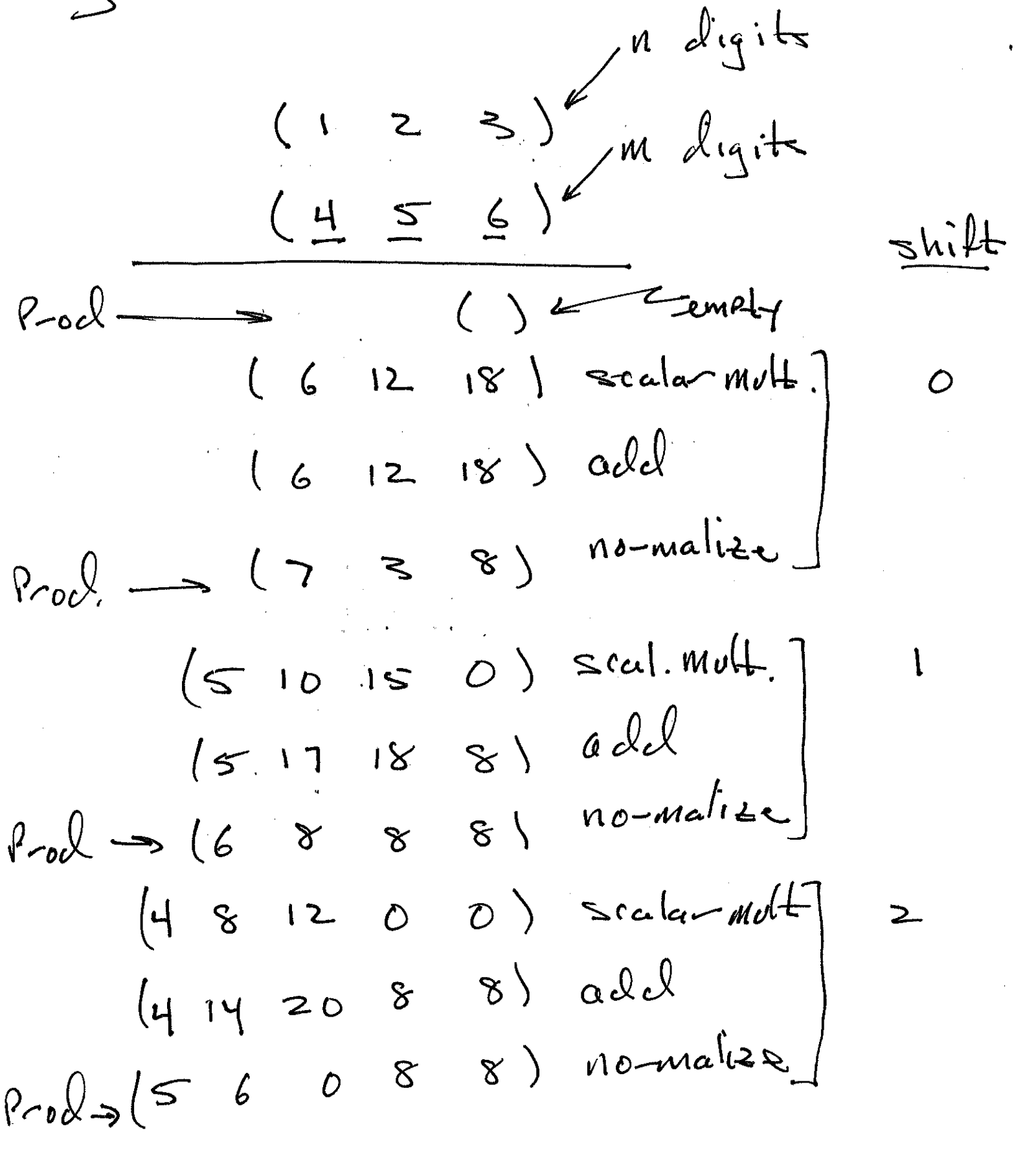
Pa6: ext. 1 day (Tues.)

Pa6 stuff:

Ex. multiplication $b=10$

$$
\begin{array}{r}
{\scriptstyle 1\ 1} \\
123 \\
\times\ 456 \\
\hline
738 \\
6150\ \cdot \\
49200 \\
\hline
56088\ \checkmark
\end{array}
$$

again:


n digits
m digits
(1 2 3)
(4 5 6)
shift
Prod → ( ) ← empty
(6 12 18) scalar mult.
(6 12 18) add
(7 3 8) no-malize
Prod. → (5 10 15 0) scal. mult.
(5 17 18 8) add
(6 8 8 8) no-malize
Prod → (4 8 12 0 0) scalar-mult
(4 14 20 8 8) add
(5 6 0 8 8) no-malize
Prod → (0
1
2

Runtime: (# digits 1st) · (# digits in 2nd)

$= \Theta(m \cdot n)$

more detail!

(# digit multiplications) $= m \cdot n$

(# digit additions) $= 2mn + m^2$

Ex. $b = 10$

$999 \times 999 = 998001$ ✓

$$\begin{array}{ccc} 9 & 9 & 9 \\ \underline{9} & \underline{9} & \underline{9} \end{array}$$

shift

Prod: (1)

81	81	81	s.m.	0
81	81	81	add	
Prod: (8	9	9	1)	norm
81	81	81	0	s.m. 1
89	90	90	1	add
Prod: (9	8	9	0	1) norm
81	81	81	0	0 s.m 2
90	89	90	0	1 add
Prod. (9	9	8	0	0 1) norm

Notice: digit arithmetic never exceeded $99 < 100 = 10^2$.

In general, in base b, this algorithm does not exceed b^2

Exercise

show in base b

$$(\underbrace{b-1, b-1, \ldots, b-1}_{n \text{ digits}})^2$$

$$= (\underbrace{b-1, \ldots, b-1}_{n-1}, b-2, \underbrace{0, \ldots, 0}_{n-1}, 1)$$

Ex. normalize $b=10$

$$\begin{matrix} -9 & & 1 & 6 & -5 & -50 & 0 \end{matrix}$$
$$(1, -90, 9, 73, 0, -500)$$

$$\begin{matrix} -8 & -89 & 15 & 68 & -50 & -500 \end{matrix}$$
$$\begin{matrix} 90 & -10 & -60 & +50 & +500 \end{matrix}$$

$$(-8 \quad 1 \quad 5 \quad 8 \quad 0 \quad 0)$$

change sign ↓

$$\begin{matrix} -1 & -1 & -1 & 0 \end{matrix}$$

$$\boxed{-1} \quad (8 \quad -1 \quad -5 \quad -8 \quad 0 \quad 0)$$

$$\begin{matrix} 7 & -2 & -6 & -8 \end{matrix}$$
$$\begin{matrix} 10 & 10 & 10 \end{matrix}$$

$$\boxed{-1} \quad (7 \quad 8 \quad 4 \quad 2 \quad 0 \quad 0)$$

Fact: $1 \cdot 10^5 + (-90) \cdot 10^4 + 9 \cdot 10^3 + 73 \cdot 10^2 + 0 \cdot 10^1 + (-500) \cdot 10^0$

$= (-1) \cdot (7 \cdot 10^5 + 8 \cdot 10^4 + 4 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10^1 + 0 \cdot 10^0)$

$= -784200$

Ex. (0, 0, 0, 0, 0) → no-make

() empty

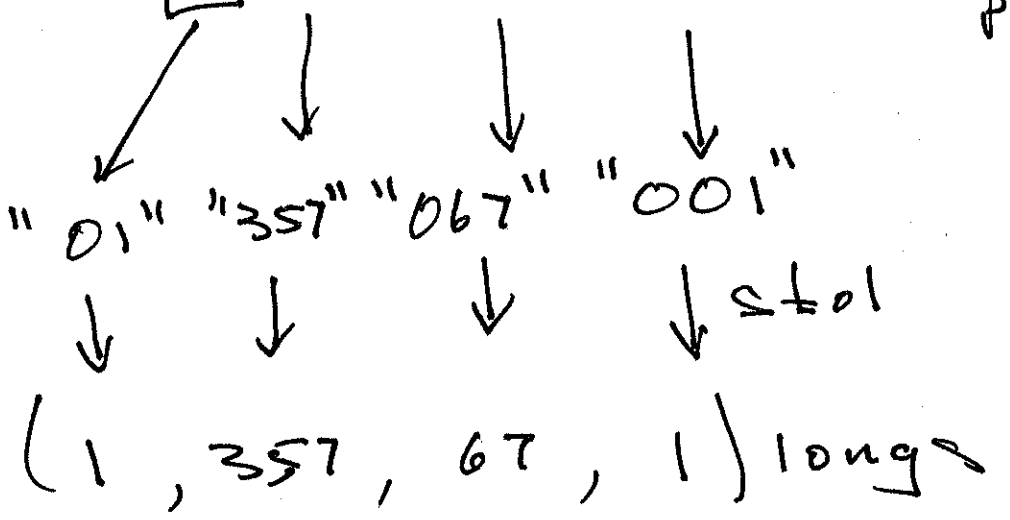
return 0

Ex. (0, 0, 3, 5, 7) →

(3, 5, 7)

constructor from string.

Ex. s = "01357067001" b = 1000, p = 3



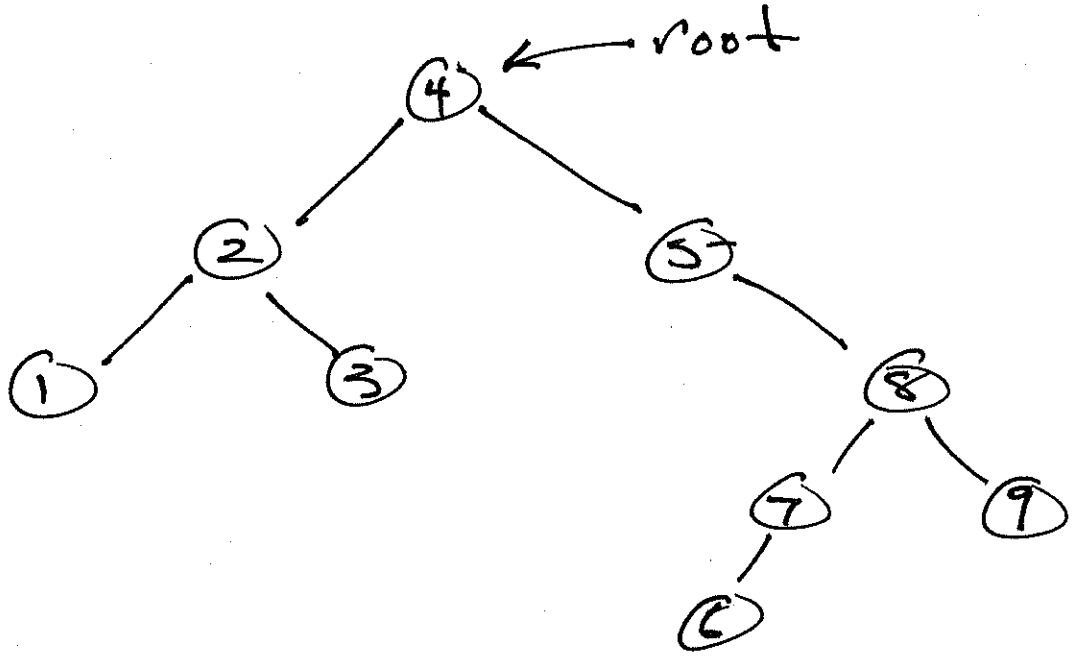
Back to BSTs

Tree traversals:

- InOrderTreeWalk(x) ← node in a BST
- PreOrderTreeWalk(x)
- PostOrderTreeWalk(x)

see Pseudo-code.

Ex.



In order : 1 2 3 4 5 6 7 8 9

Pre order : 4 2 1 3 5 8 7 6 9

Post order : 1 3 2 6 7 9 8 5 4