

- Quiz 2 Today
- Pas ext. 2 days
- Pa 4 posted.

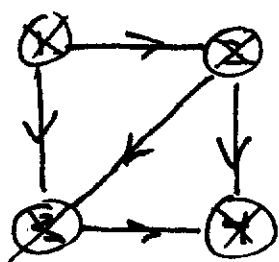
How to use DFS to obtain a
topological sort of a DAG G ,

- Execute DFS(G)
- as vertices finish, push them
onto a stack S . (equivalently:
sort vertices by dec. finish time.)

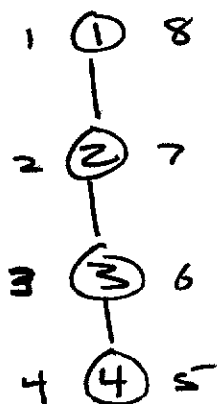
Then when done, S is a topological
sort of $V(G)$.

Ex.

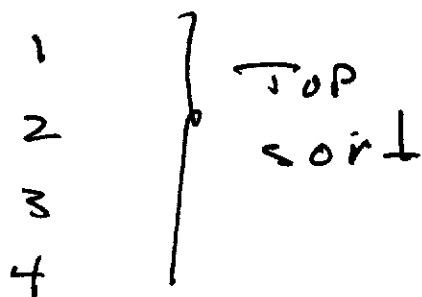
G



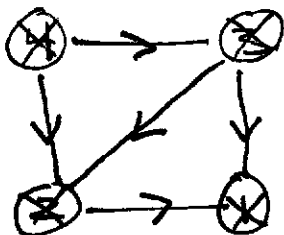
Forest:



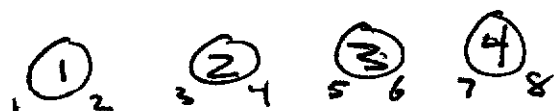
Stack



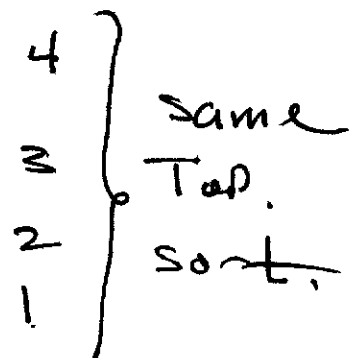
Ex.



Forest:

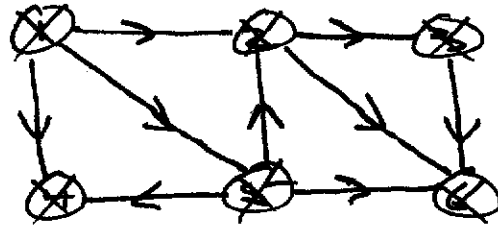


stack

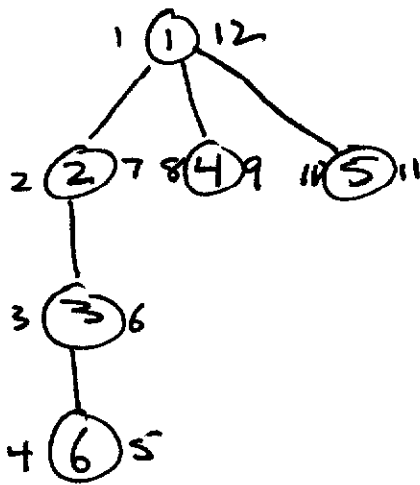


Ex.

G

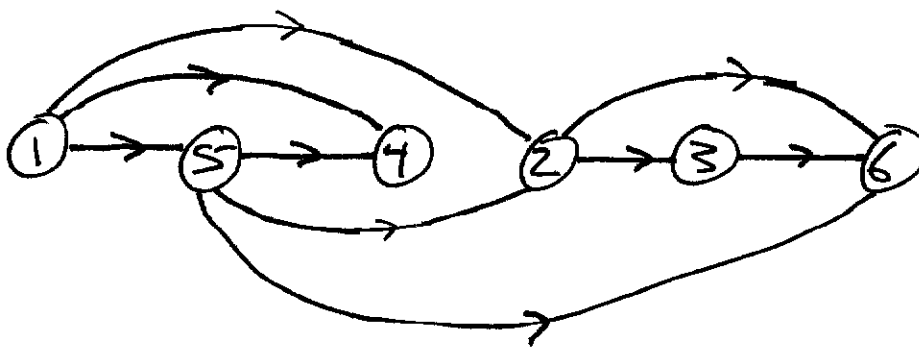


Forest



Stack

1
5
4
2
3
6



Topological sort of $V(G)$

Exercise:

write down 3 other topological sorts

Exercise

How can you sort an array using topological sort.

Strongly Connected Components

let $G = (V, E)$ be a digraph.

We say y is reachable from x

iff G contains a directed x - y Path



Defn

we say G is strongly connected

iff every vertex in G is reachable from every other vertex.

more generally, a subset $U \subseteq V(G)$

is said to be strongly connected

iff for all $x, y \in U$, x is reachable from y and y is reachable from x .

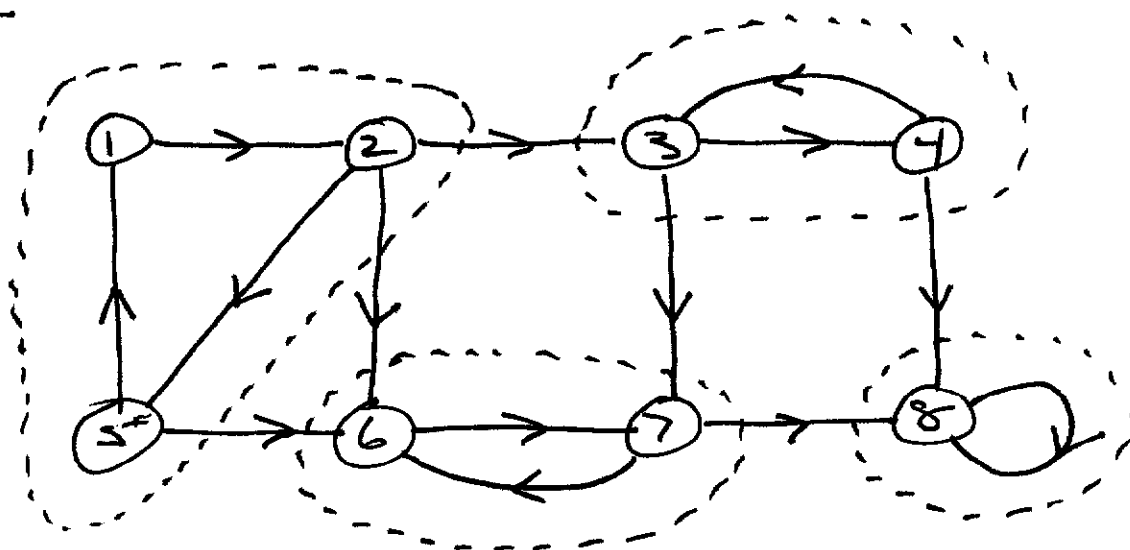
Also $U \subseteq V(G)$ is called a strongly connected component of G iff

(1) U is strongly connected, and

(2) U is maximal w.r.t. (1).

(2) means that if $U \subsetneq W \subseteq V$,
then W is not strongly connected.

Ex.



strong components (SCCs) !

$\{1, 2, 5\}$

$\{3, 4\}$

$\{6, 7\}$

$\{8\}$

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To find the SCCs of a digraph G :

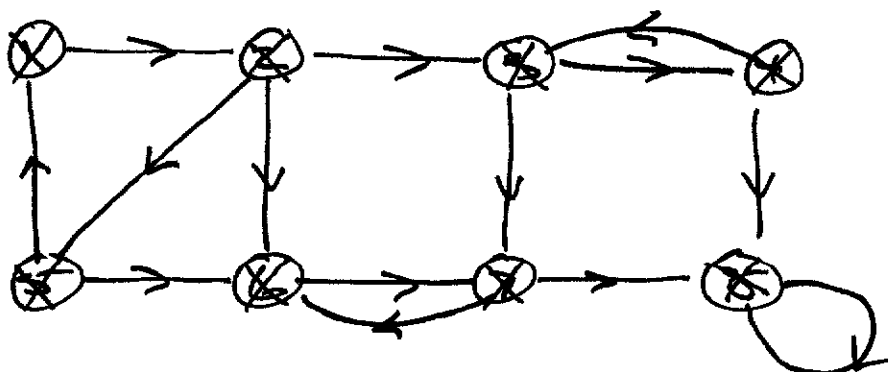
- Run DFS(G), as vertices finish Push them onto a stack S .
- Compute transpose G^T of G .
(reverse all directed edges)
- Run DFS(G^T), Process main loop of DFS by dec. finish times from 1st call, i.e. Pop vertices off the stack.

Theorem

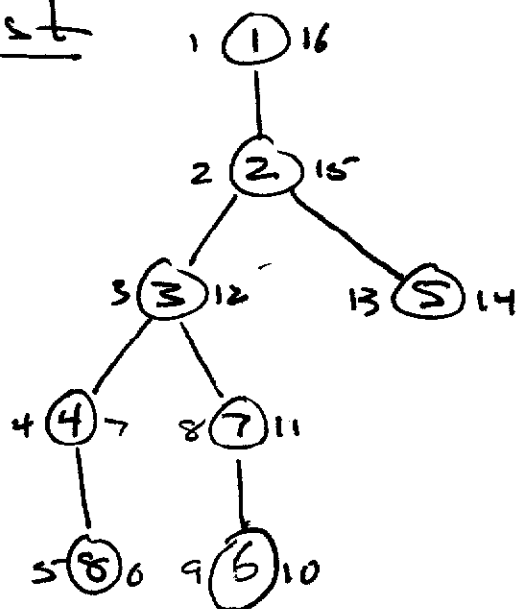
when this is done, the vertex sets of the trees in the 2nd DFS Forest are the SCCs of G ,

Ex.

G



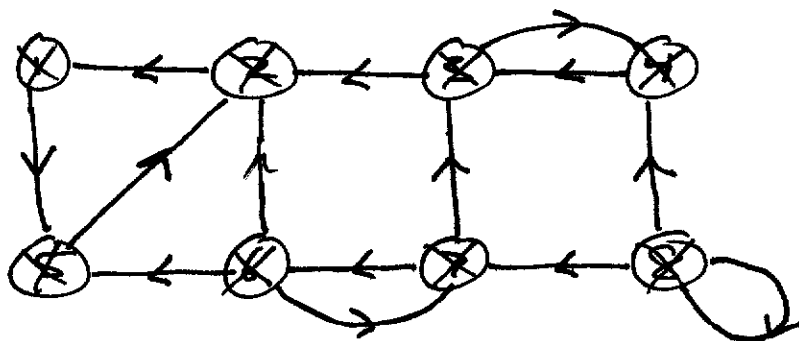
Forest



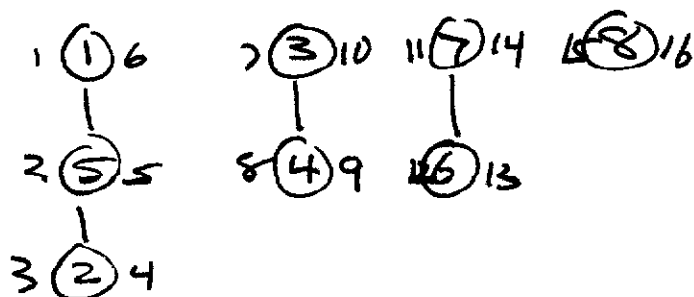
stack

1	✓
2	✓
5	✓
3	✓
7	✓
6	✓
4	✓
8	✓

G^T



Forest



stack Par

8	n
7	n
6	
3	n
4	
1	n
5	
2	

SCC's of G are trees:

$$C_1 = \{1, 2, 5\}$$

$$C_2 = \{3, 4\}$$

$$C_3 = \{6, 7\}$$

$$C_4 = \{8\}$$

Notice: The SCCs of G and G^T are one and the same.

Defn

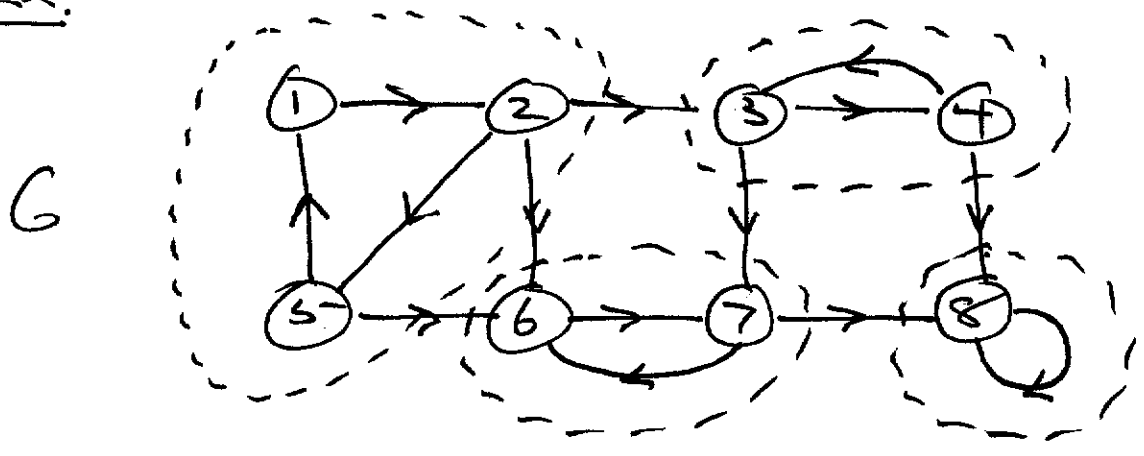
The component Graph (also condensation Graph) of G , denoted G^{SCC} is the digraph with

$$V(G^{SCC}) = \{SCCs \text{ of } G\}$$

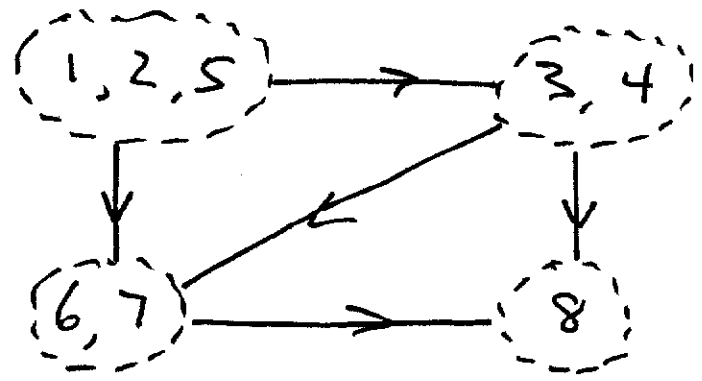
and

$$E(G^{scc}) = \left\{ (C_i, C_j) \mid \exists x \in C_i, y \in C_j \text{ and } (x, y) \in E(G) \right\}$$

Ex.



G^{scc}



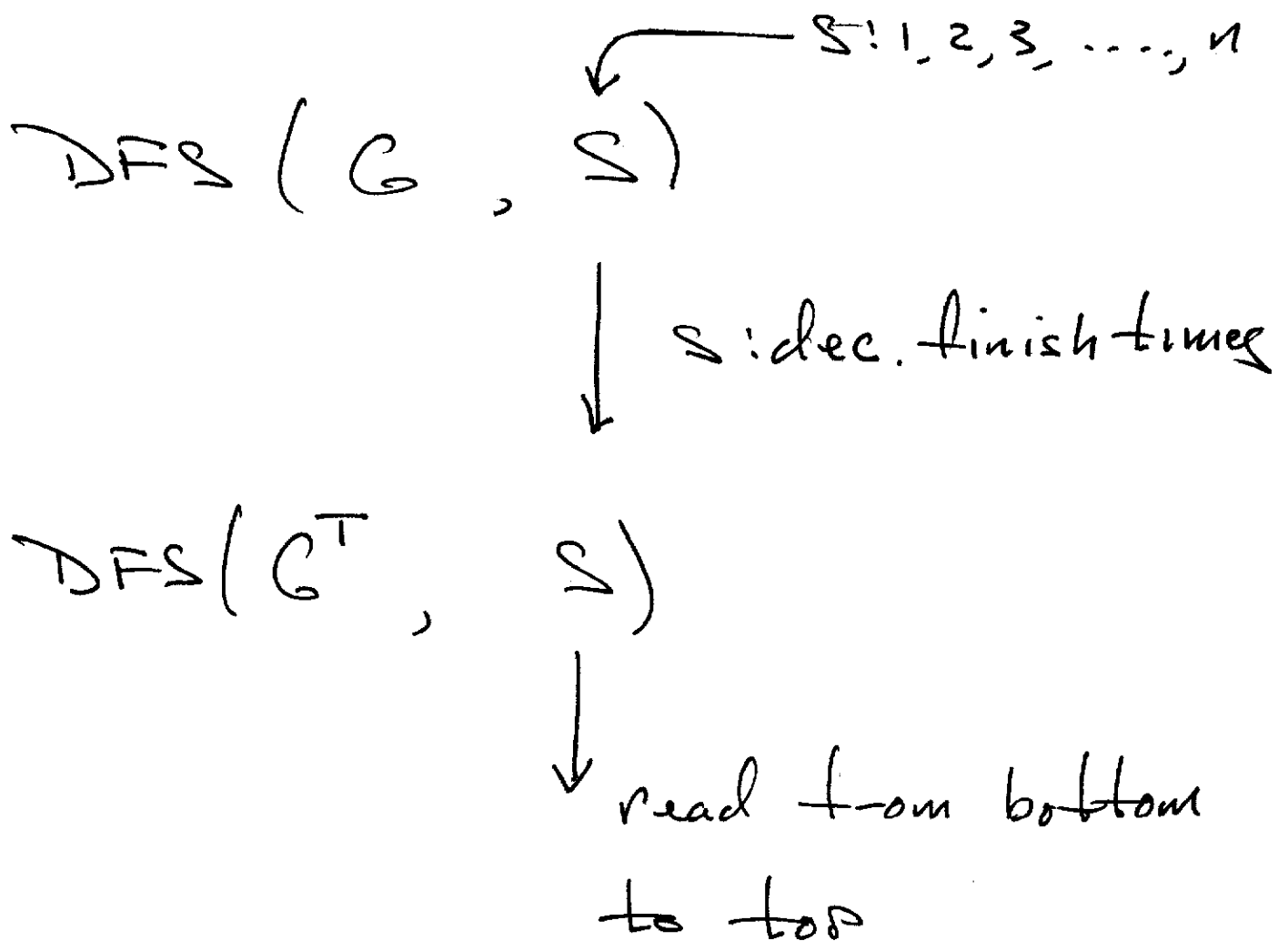
notice: G^{scc} is a DAG.

It has a topological sort.

Pa3

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To find SCC's after 2nd call
read stack from bottom to
top. Also gives SCC's in topologically
sorted order.



Output file

Component 1 : 1 5 2

Component 2 : 3 4

Component 3 : 7 6

Component 4 : 8

S is Both list controlling main loop of DFS and Stack recording finish times. How?

