CSE 101
Winter 2021
Quiz 4 **Solutions**

1. (25 Points) Insert the keys: 5, 9, 7, 2, 6, 4, 8, 3, 1, 10 (in order) into an initially empty Binary Search Tree T.

   a. (5 Points) Give the keys in the order printed by a **pre-order tree walk**.

      Solution: 5, 2, 1, 4, 3, 9, 7, 6, 8, 10

   b. (5 Points) Give the keys in the order printed by a **post-order tree walk**.

      Solution: 1, 3, 4, 2, 6, 8, 7, 10, 9, 5

   c. (5 Points) Is it possible to assign the colors {Red, Black} to the vertices of T so that the Red-Black Tree properties (see notes 2-16-21 p. 7) are satisfied, and $\mathrm{bh}(T) = 1$? If it is possible, specify all such colorings by stating, for each coloring, the set of keys belonging to red nodes.

      Solution: No such color assignment is possible.

   d. (5 Points) Is it possible to assign the colors {Red, Black} to the vertices of T so that the Red-Black Tree properties (see notes 2-16-21 p. 7) are satisfied, and $\mathrm{bh}(T) = 2$? If it is possible, specify all such colorings by stating, for each coloring, the set of keys belonging to red nodes.

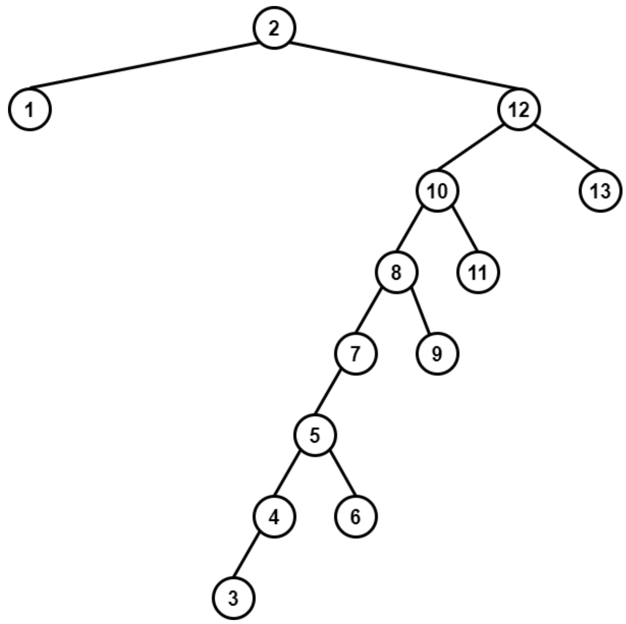
      Solution: There is one way to make $\mathrm{bh}(T) = 2$.
      - $\text{Red} = \{2, 3, 6, 8, 9\}$

   e. (5 Points) Is it possible to assign the colors {Red, Black} to the vertices of T so that the Red-Black Tree properties (see notes 2-16-21 p. 7) are satisfied, and $\mathrm{bh}(T) = 3$? If it is possible, specify all such colorings by stating, for each coloring, the set of keys belonging to red nodes.

      Solution: There are two ways to make $\mathrm{bh}(T) = 3$.
      - $\text{Red} = \{3, 7\}$
      - $\text{Red} = \{3, 6, 8\}$

(See next page for problem 2)

2. (25 Points) Consider the Binary Search Tree pictured below.



Use the Theorem discussed in class (see notes 2-18-21 page 1) to explain why it is **impossible** to assign colors Red and Black to the nodes in this tree so as to satisfy the RBT properties. (Note: be sure to include the nil leaves when calculating the height of this tree.)

Solution:
According to the stated theorem, we must have $h \leq 2\lg(n+1)$ for any RBT with n internal nodes and (absolute) height h. Observe that the tree above has $n = 13$ and $h = 8$ (counting the nil children of node 3 as the deepest leaves.) But then

$$2\lg(n+1) = 2\lg(14) < 2\lg(16) = 2 \cdot 4 = 8 = h$$

making the inequality in the theorem is false. Therefore above tree cannot be colored so as to satisfy the RBT properties.