

CSE 101

Introduction to Data Structures and Algorithms

Programming Assignment 7

In this assignment you will create a BigInteger ADT capable of performing arithmetic operations on arbitrarily large signed integers. The underlying data structure for this ADT will be a List of longs. For this purpose, it will be necessary to alter your List ADT from pa1 slightly. This entails changing certain field types, declaration statements, method parameters, and return types from `int` to `long`. I suggest that you use `#define` to create a constant macro for your list element type as follows.

```
#define LIST_ELEMENT long
```

Place this line at the beginning of List.h, then insert the macro `LIST_ELEMENT` in place of `int` in the appropriate places in List.c. This way it will be easy to change the list type again if you need to.

It will also be necessary to rename or remove function `equals()` from the List ADT. This is because that name will be used as a required function in the BigInteger ADT. Once these small changes are made, thoroughly test the new version of List with your own ListTest client.

The BigInteger ADT will represent a signed integer by encapsulating two pieces of data: an `int` (which will be either 1, -1, or 0) giving its sign, and a List of non-negative longs representing its magnitude. Each List element will be a single digit in the base b positional numbering system, where b is a power of 10: $b = 10^p$. For reasons that will become clear, we restrict the exponent p to the range $1 \leq p \leq 9$. The BigInteger arithmetic operations will utilize the `long` arithmetic built into the C language to perform operations on single (base b) digits, and build up the standard arithmetic operations (add, subtract, multiply) out of these (base b) digit operations. The reason we chose b to be a power of 10 is to facilitate the conversion between base 10 and base b . In the case $p = 2$, we have $b = 100$, each base b digit consists of 2 base 10 digits.

Base 100 digits = {00, 01, 02,, 97, 98, 99}

The 52-digit base 10 number

$N = 6523485630758234007488392857982374523487612398700554,$

(too large for any built-in C data type) becomes the following List of 26 base 100 digits.

$L = (65\ 23\ 48\ 56\ 30\ 75\ 82\ 34\ 00\ 74\ 88\ 39\ 28\ 57\ 98\ 23\ 74\ 52\ 34\ 87\ 61\ 23\ 98\ 70\ 05\ 54)$

where we have separated digits by a space for the sake of readability. The same number in base 1,000 (i.e. $p = 3$) would have 18 digits:

$L = (006\ 523\ 485\ 630\ 758\ 234\ 007\ 488\ 392\ 857\ 982\ 374\ 523\ 487\ 612\ 398\ 700\ 554),$

and in base 1,000,000,000 ($p = 9$) it would have 6 digits:

$L = (006523485\ 630758234\ 007488392\ 857982374\ 523487612\ 398700554).$

Observe that to print out the base 10 representation of such a number, we need only concatenate the base 10 digits of each of its base b digits, then strip off any leading zeros. To go the opposite direction and parse

a string of base 10 digits into a List of base b digits, we separate the string into groups of p characters, working from right to left. The final, leftmost, base b digit may be parsed from fewer than p characters.

In all of these Lists, we regard the front as being the right end, and the back as the left. With this convention, the List index of a base b digit is the corresponding power on b . Thus

$$L = (c_{n-1} \ c_{n-2} \ \cdots \ c_2 \ c_1 \ c_0)$$

represents the number

$$N = c_{n-1}b^{n-1} + c_{n-2}b^{n-2} + \cdots + c_2b^2 + c_1b^1 + c_0b^0.$$

It is instructive at this point to do some arithmetic examples in such a representation. To illustrate, we take $p = 2$, so $b = 100$.

Example

carry:	1 1	-1
	(35 57 97)	(35 57 97)
	<u>+ (14 90 82)</u>	<u>- (14 90 82)</u>
	(50 48 79)	(20 67 15)

As we can see, it is very useful to think of a borrow in subtraction as nothing more than a negative carry. One checks easily that in base 10: $355797 + 149082 = 504879$ and $355797 - 149082 = 206715$. Another way to do these examples is to add and subtract Lists as vectors, then "normalize" the results, by working from right to left in each vector, carrying and borrowing as needed to obtain a List of longs in the range $0 \leq x < b$, i.e. base b digits.

Example

	(35 57 97)	(35 57 97)
	<u>+ (14 90 82)</u>	<u>- (14 90 82)</u>
	(49 147 179)	(21 -33 15)
carry:	1 1 0	-1 0 0
normalize:	(49 147 179)	(21 -33 15)
	-100 -100	100
result:	(50 48 79)	(20 67 15)

The reader is urged at this point to do a large number of such examples, since these are exactly the algorithms needed to perform BigInteger addition and subtraction. The subtraction example above (in particular the value -33) illustrates why it is useful to use `long` instead of `unsigned long` as our List element data type. It allows us to do signed arithmetic at the level of each base b digit. You should also attempt to do some multiplication problems in this representation. In these examples, it is necessary for each digit in the first BigInteger to multiply each digit in the second BigInteger. If x and y are two such digits, then $0 \leq x < b$ and $0 \leq y < b$, hence $0 \leq xy < b^2$. In order to guarantee that this product is always computable in type `long`, we must have b^2 no larger than the maximum possible `long` value, i.e.

$$b^2 \leq 2^{63} - 1$$

and therefore

$$b \leq \sqrt{2^{63} - 1} = 3,037,000,499.97 \dots$$

The largest such b that is also a power of 10 is $b = 1,000,000,000 = 10^9$, which explains why the power p on 10 must be in the range $1 \leq p \leq 9$.

BigInteger ADT Specifications

The BigInteger ADT will be implemented in files BigInteger.h and BigInteger.c. Following our standard practice BigInteger.c will contain a typedef for BigIntegerObj and BigInteger.h will define the reference type BigInteger as a pointer to BigIntegerObj. It is recommended that BigInteger.c contain either global constants or #define constant macros for both BASE and POWER satisfying $\text{BASE} = 10^{\text{POWER}}$. During your testing phase, you may have any value $0 \leq \text{POWER} \leq 9$, but when you submit your project, be sure to set $\text{POWER} = 9$ and $\text{BASE} = 1000000000$, for proper grading. The BigIntegerObj should have at least two components: an int (1, -1 or 0) specifying the sign, and a normalized List of longs specifying the magnitude. (Here "normalized" means each List element x is in the range $0 \leq x < \text{BASE}$.) Your BigInteger ADT will implement the following operations and types.

```
// Exported type -----

// BigInteger reference type
typedef struct BigIntegerObj* BigInteger;

// Constructors-Destructors -----

// newBigInteger()
// Returns a reference to a new BigInteger object in the zero state.
BigInteger newBigInteger();

// freeBigInteger()
// Frees heap memory associated with *pN, sets *pN to NULL.
void freeBigInteger(BigInteger* pN);

// Access functions -----

// sign()
// Returns -1 if N is negative, 1 if N is positive, and 0 if N is in the zero
// state.
int sign(BigInteger N);

// compare()
// Returns -1 if A<B, 1 if A>B, and 0 if A=B.
int compare(BigInteger A, BigInteger B);

// equals()
// Return true (1) if A and B are equal, false (0) otherwise.
int equals(BigInteger A, BigInteger B);

// Manipulation procedures -----

// makeZero()
// Re-sets N to the zero state.
void makeZero(BigInteger N);
```

```

// negate()
// Reverses the sign of N: positive <--> negative. Does nothing if N is in the
// zero state.
void negate(BigInteger N);

// BigInteger Arithmetic operations -----

// stringToBigInteger()
// Returns a reference to a new BigInteger object representing the decimal integer
// represented in base 10 by the string s.
// Pre: s is a non-empty string containing only base ten digits {0,1,2,3,4,5,6,7,8,9}
// and an optional sign {+, -} prefix.
BigInteger stringToBigInteger(char* s);

// copy()
// Returns a reference to a new BigInteger object in the same state as N.
BigInteger copy(BigInteger N);

// add()
// Places the sum of A and B in the existing BigInteger S, overwriting its
// current state: S = A + B
void add(BigInteger S, BigInteger A, BigInteger B);

// sum()
// Returns a reference to a new BigInteger object representing A + B.
BigInteger sum(BigInteger A, BigInteger B);

// subtract()
// Places the difference of A and B in the existing BigInteger D, overwriting
// its current state: D = A - B
void subtract(BigInteger D, BigInteger A, BigInteger B);

// diff()
// Returns a reference to a new BigInteger object representing A - B.
BigInteger diff(BigInteger A, BigInteger B);

// multiply()
// Places the product of A and B in the existing BigInteger P, overwriting
// its current state: P = A*B
void multiply(BigInteger P, BigInteger A, BigInteger B);

// prod()
// Returns a reference to a new BigInteger object representing A*B
BigInteger prod(BigInteger A, BigInteger B);

// Other operations -----

// printBigInteger()
// Prints a base 10 string representation of N to filestream out.
void printBigInteger(FILE* out, BigInteger N);

```

The zero state will be represented by the sign 0 and an empty List. A positive integer will be represented by the sign 1, and a non-empty List containing the (normalized) base b digits of its magnitude. A negative integer will be represented by the sign -1 and a non-empty List containing the (normalized) base b digits of its magnitude. Notice that each arithmetic operation has two functions. The *verbs* `add()`, `subtract()` and `multiply()` and the *nouns* `sum()`, `diff()` and `prod()`. The verbs are each defined to take a first argument which is to be overwritten by the result of operating on the other two arguments. Therefore

`add(S, A, B)` essentially performs the assignment $S = A + B$. These operations must be defined in such a way that different arguments may refer to one and the same `BigInteger` object. Thus it must be possible to assign $S = S + A$, or $S = A + S$, or even $S = S + S$, without inadvertently altering the object S during the computation. Similar comments go for the other verbs `subtract()` and `multiply()`. The nouns `sum()`, `diff()` and `prod()` are essentially constructors, since they return references to newly allocated `BigInteger` objects.

Notice that one important arithmetic operation has been left out of this exercise, namely integer division (with two outputs: quotient and remainder.) You may include functions implementing this operation if you wish, but your grade will in no way depend on it. A judiciously chosen set of private helper functions will be very useful in this project. It is suggested that such a function to "normalize" a List of longs be included in the set.

Client Module

You will write a top level client called `Arithmetic.c` that uses the exported operations in `BigInteger.c`. This client will read from an input file containing exactly 4 lines:

- line 1: a positive integer a
- line 2: an optional sign character (+ or −) followed by a decimal digits
- line 3: a positive integer b
- line 4: an optional sign character (+ or −) followed by b decimal digits

`Arithmetic.c` will open such an input file, and an output file (to be overwritten if it already exists). It will parse the string on line 2 as a `BigInteger` A , and the string on line 4 as a `BigInteger` B . It will then compute the following quantities and write their base 10 digits (including a sign in the negative case) to the output file.

$$A, B, A + B, A - B, A - A, 3A - 2B, AB, A^2, B^2, 9A^4 + 16B^5$$

These quantities will be printed in the above order, each on its own line, separated by blank lines. A set of matched input and output files will be posted in the Examples section of the webpage for testing purposes.

What to Turn In

You will also write your own test clients for both the List and `BigInteger` ADTs. As usual, these files are your scratch work, showing the tests you performed in building the two modules. Additional test clients may be posted on the webpage as (weak) tests of the two ADTs. These files are for your edification only and are not to be submitted. You will submit the following 9 files in all.

<code>Arithmetic.c</code>	Top level client for the project, described above
<code>BigInteger.c</code>	<code>BigInteger</code> implementation file
<code>BigInteger.h</code>	<code>BigInteger</code> header file
<code>BigIntegerTest.c</code>	Your test suite for the <code>BigInteger</code> ADT
<code>List.c</code>	List implementation file
<code>List.h</code>	List header file
<code>ListTest.c</code>	Your test suite for the List ADT
<code>README</code>	Content listing of the project
<code>Makefile</code>	Included in Examples, alter as you see fit

This project is quite challenging, and should be started as early as possible. As usual, formulate questions and get help as needed.