CSE 101   3-4-21

PA7: normalize() function (helper)

subtraction.

Ex. $A = (25, 13, 78, 1)$ $b = 100$, $p = 2$

$B = (48, 22, 31, 12)$ A − B

−1 −1 −1 −1

$(-23, -9, 47, -11)$ normalize

−1 −1 −24 +100

+100 +100 +100 +100

------- 99, 76, 91, 46, 89)

stop when leftmost digit is negative.

$(-24, 91, 46, 89)$ change all signs

$(-1) \times (24, -91, -46, -89)$ normalize again.

$$(\overset{-1}{24}, \overset{-1}{-91}, \overset{-1}{-46}, -89)$$

$$\begin{array}{ccc} -92 & -47 & \\ +100 & +100 & +100 \end{array}$$

$$(-1) \times \underbrace{(23, 08, 53, 11)}_{}$$

return this sign

normalized list

<u>check</u>:

$$\begin{array}{r} 25137801 \\ -\ 48223112 \\ \hline -\ 23085311 \end{array}$$

<u>Ex</u>. same, addition

$$\left.\begin{array}{l} A = (25, 13, 78, 01) \\ B = (48, 22, 31, 12) \end{array}\right] A+B$$

$$A+B = (73, 35, 109, 13)$$

normalize:

$$(73, \overset{+1}{\underset{36}{35}}, \underset{-100}{109}, 13)$$

$$(+1) \times (\underbrace{73, 36, 09, 13}_{\text{normalized list}})$$

return this sign.

<u>Ex.</u> $A = (89, 13, 58, 91)$

$B = (95, 80, 60, 23)$

$A+B = (184, 93, 118, 114)$

normalize:

$$\begin{array}{cccc} {}^{+1} & & {}^{+1} & {}^{+1} \\ (184, & 93, & 118, & 114) \\ 01 & 94 & 119 & \\ -100 & & -100 & -100 \end{array}$$

$(1, 84, 94, 19, 14)$

check:

$$\begin{array}{r} 89135891 \\ +\ 95806023 \\ \hline 184941914 \end{array}$$

Back to SSSP ......

Both Dijkstra and Bellman-Ford use vertex attributes:

$P[x]$: Parent of x, encodes the shortest paths tree (or predecessor subgraph.) use PrintPath(s, x) to print min weight s-x path

$d[x]$: estimate of $\delta(s,x)$.

Predecessor subgraph : $G_p = (V_p, E_p)$

$V_p = \{ x \in V \mid P[x] \neq nil \text{ or } x = s \}$ vertices reachable from s.

$E_p = \{ (P[x], x) \mid P[x] \neq nil \}$

See PrintPath(G, s, x) p. 601 sec. 22.2

Helper functions:

Initialize(G, s)

```
for all x ∈ V
    d[x] = ∞
    p[x] = nil
d[s] = 0
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Relax(x, y) pre: $y \in adj[x]$.

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if d[y] > d[x] + w(x, y)
    d[y] = d[x] + w(x, y)
    p[y] = x
```

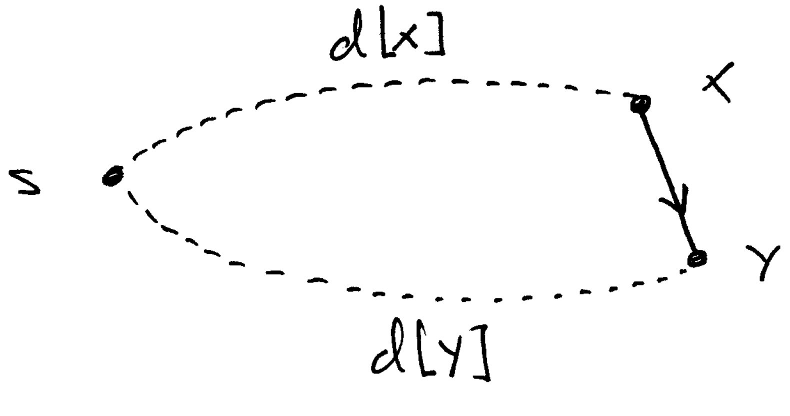

Rmks:

- Relax(x,y) changes only attributes of y.
- After Relax(x,y), the inequality

$$d[y] \leq d[x] + w(x,y)$$

is necessarily true.

Three lemmas

Lemma 1:

Let $x \in V(G)$ and suppose that after Initialize(G,s), some sequence of calls to Relax$(\cdot,\cdot)$ results in $d[x] < \infty$. Then G contains an s-x path of weight $d[x]$.

Lemma 2

After Initialize(G, s), the inequality

$$\delta(s, x) \leq d[x] \quad (\forall x \in V(G))$$

is maintained over any seq. of calls to Relax$(\cdot,\cdot)$.

Lemma 3 (Path relaxation Property)

If $P: s = v_0, v_1, v_2, \ldots, v_k = x$ is a min-weight s-x Path in G, and the edges of P are relaxed in order

$$(\underset{s}{v_0}, v_1), (v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, \underset{x}{v_k}),$$

then $d[x] = \delta(s, x)$. This is true regardless of any other relaxations that occur, even if interspersed with the above relaxations.

- see BellmanFord(G, s)
- see Dijkstra(G, s)

<u>Pay again ------.</u>

$$(\cancel{0}, \cancel{0}, \cancel{0}, 35, 14) \xrightarrow{\text{normalize()}} (35, 14)$$

$$(0, 0, 0, 0, 0) \longrightarrow ()$$ return sign 0.

multiplication!

$$
\begin{array}{rl}
 & (23, 45, 15) \\
\times & (71, 08, 22)
\end{array}
$$

$$(\cdot , \cdot , \cdot)$$

$$(\cdot , \cdot , \cdot , \cdot)$$

$$(\cdot , \cdot , \cdot , \cdot , \cdot)$$

helper function:

long

list of longs

scalarMultiplyList(m, L)