CSE 101 3-11-21

- Final: Tue Mar. 16 6am-2PM
- SETs: Due Sun. Mar 14 11:59 PM

Chap 23: (MWST)
minimum weight spanning trees

Defn
A spanning tree in a graph $G=(V,E)$ is a subgraph T of G satisfying

(1) T is a tree, and

(2) $V(T)=V(G)$.

Defn

$$w(T)=\sum_{e\in E(T)} w(e)$$

Defn a MWST in G is a sp. tree T such that

$$w(T) \le w(S)$$

for all sp. trees S in G.

Rmk
G contains a sp. tree iff G is connected.

Algorithms:

- Prim ←
- Kruskal ←
- Dual-Kruskal

} ch. 23 of CLRS.

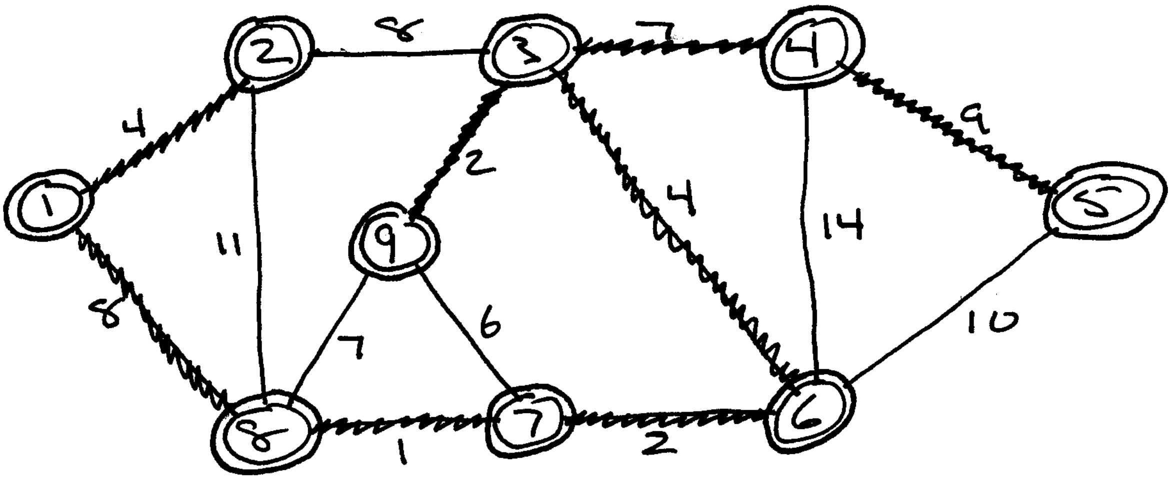
Prim(G)

1. Pick any $r \in V$, set $W = \{r\}$
2. $F = \phi$
3. while $|F| < n-1$ // $n = |V|$
4. amongst all edges in $E-F$ with one end in W and other in $V-W$, let e be one of min. weight.
5. $F = F \cup \{e\}$
6. $W = W \cup \{$other end of $e\}$

Thm

when execution is complete, $W = V$, and $T = (V, F)$ is a MWST in G.

Ex. $r = 3$



$T = (V, F)$

$F = \{12, 18, 34, 36, 39, 45, 67, 78\}$

$\text{weight}(T) = 4 + 8 + 7 + 4 + 2 + 9 + 2 + 1$

$= \boxed{37}$

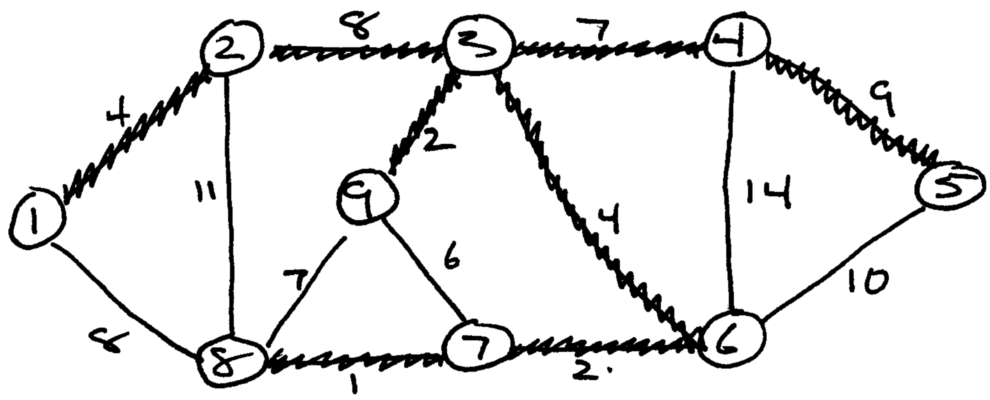
Kruskal (G)

1. $F = \emptyset$
2. while $|F| < n-1$ // $n = |V|$
3. amongst all edges in $E - F$ whose addition to F would not create a cycle, let e be one of min. weight
4. $F = F \cup \{e\}$

Thm

when execution completes, $T = (V, F)$ is a mwst in G.

Ex.



$T = (V, F)$

$F = \{12, 23, 34, 45, 39, 36, 37, 78\}$

$\text{weight}(T) = \ldots\ldots = \boxed{37}$

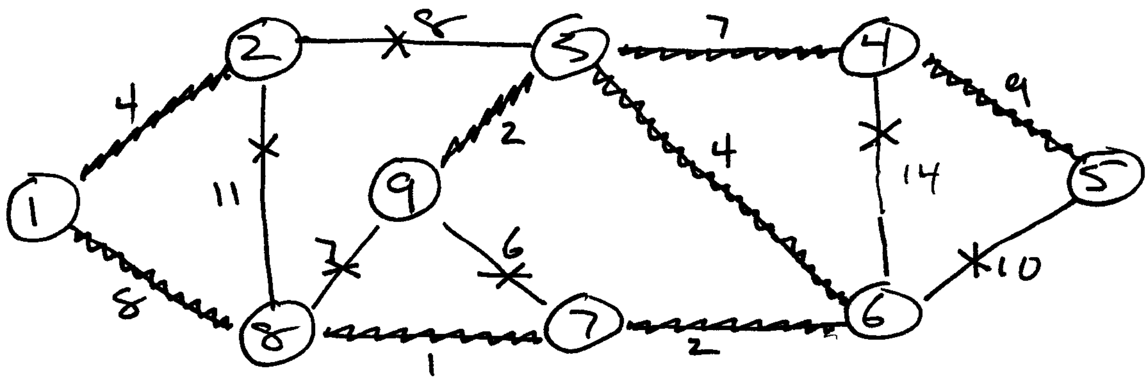
DualKruskal(G)

1. $F = E$
2. while $|F| > n-1$
3. amongst all edges in F whose removal from F does not disconnect (F, V), let e be one of max. weight.
4. $F = F - \{e\}$

Thm

When execution is complete, $T = (V, F)$ is a min weight sp. tree in G.

Ex.



$T = (V, F)$

$F = \{12, 18, 34, 36, 39, 45, 67, 78\}$

$\text{weight}(T) = \ldots\ldots = \boxed{37}$

<u>See</u>: Disjoint set data structures
Chap. 21 in CLRS

ADT: Disjoint set

- state $\mathcal{S} = \{ \{ .. \}, \{ ... \}, \ldots, \{ ... \} \}$

  disjoint

- operations
  - makeSet(x): creates $\{x\} \in \mathcal{S}$
  - FindSet(x): returns rep. of set containing x
  - union(x, y): joins sets containing x and y.