

Pass : one more day (really last!)

Thm

if T is a RBT with n (internal) nodes and height h , then

$$h \leq 2 \lg(n+1).$$

Ex Previous : $n=12$, $h=5$

$$2 \lg(n+1) = 2 \lg(13) > 6 > 5 = h$$

$$\therefore h \leq 2 \lg(n+1)$$

Ex. Previous-previous : $n=8$, $h=8$

$$2 \lg(n+1) = 2 \lg(9) = 6.33.. < 8 = h$$

$\therefore h \neq 2 \lg(n+1)$, $\therefore T$ is not a RBT,
i.e., it is impossible to assign colors to
nodes in T satisfying RBT Properties.

note:

Insert() and Delete() for RSTs
do not preserve RRT Properties.

However, they can be altered
so as to preserve RRT Properties,
and still run in time

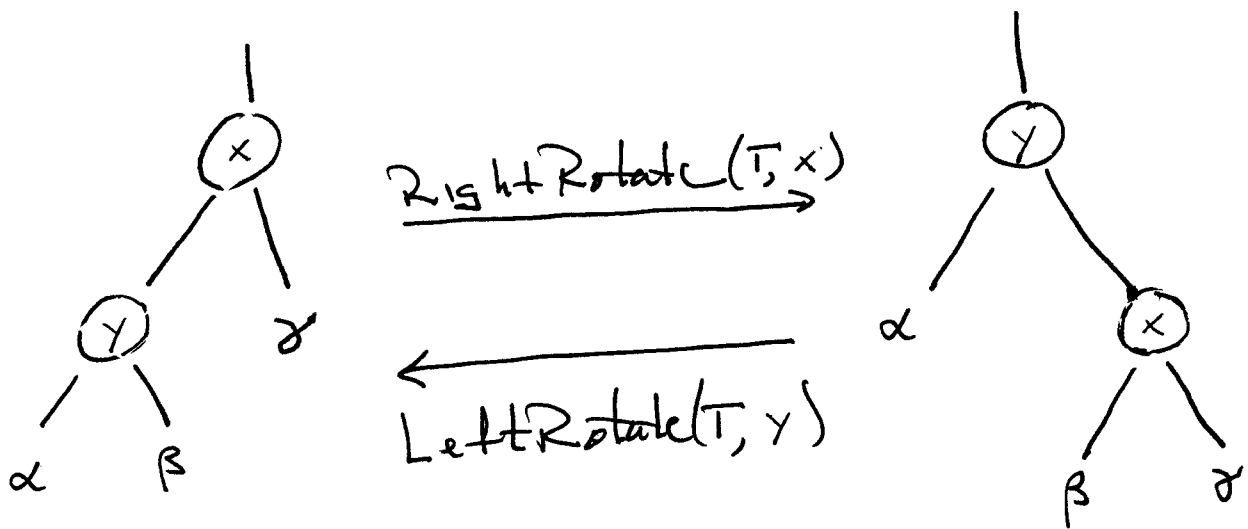
$$\Theta(\text{height}(T)) = \Theta(\log n)$$

sec. 13.2 Rotation

Rotation is a tool for altering
a tree that does not satisfy RRT
Properties, into one that does.

- LeftRotate()
 - RightRotate()
- } do not preserve RRT Prop.

Picture:



Summarize $\text{Right Rotate}(T, x)$

$$\beta \left\{ \begin{array}{l} \bullet \text{ left}[x] = \text{right}[y] \\ \bullet p[\text{right}[y]] = x \end{array} \right.$$

$$\text{Parent} \left\{ \begin{array}{l} \bullet p[y] = p[x] \\ \bullet p[x]'s \text{ (left or right) child} = y \end{array} \right.$$

$$x, y \text{ relation} \left\{ \begin{array}{l} \bullet \text{right}[y] = x \\ \bullet p[x] = y \end{array} \right.$$

cost: $\Theta(1)$.

13.3 Insertion in a RBT

4

- color node z to be inserted Red, then do BST insert
- RBT Prop. 5 is preserved.
- what about 1-4?

1. ✓

2. ✓

3. color root Black. ✓

→ 4. Red must have 2 Black children, i.e. no two Reds in a row.

└ could be a problem.

• $RB\text{-Insert}(T, z)$

• $RB\text{-Insert-Fixup}(T, z)$

RR_InsertFixup(T, z) !

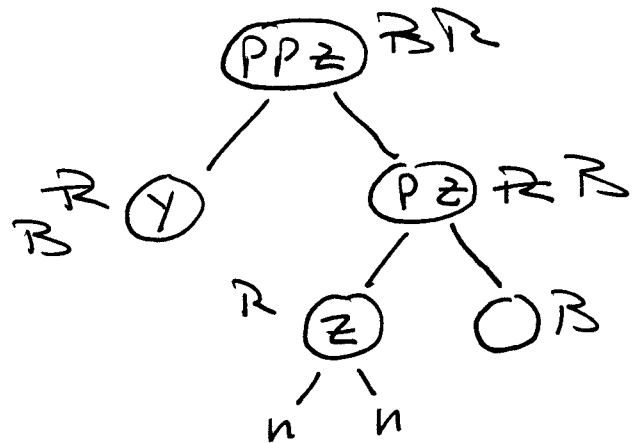
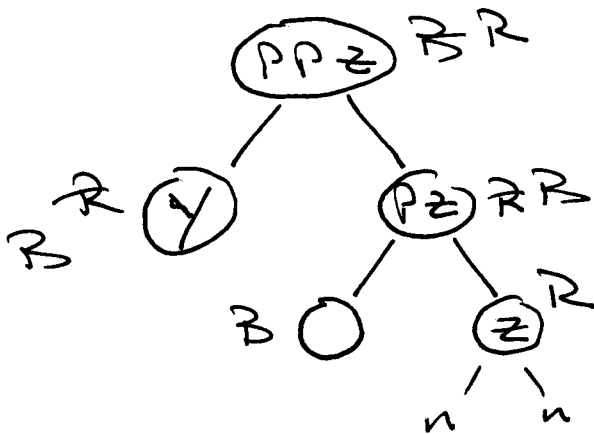
Cases 4, 5, 6 :

if $z.Parent == z.Parent.Parent.right$

$y = z.Parent.Parent.left$

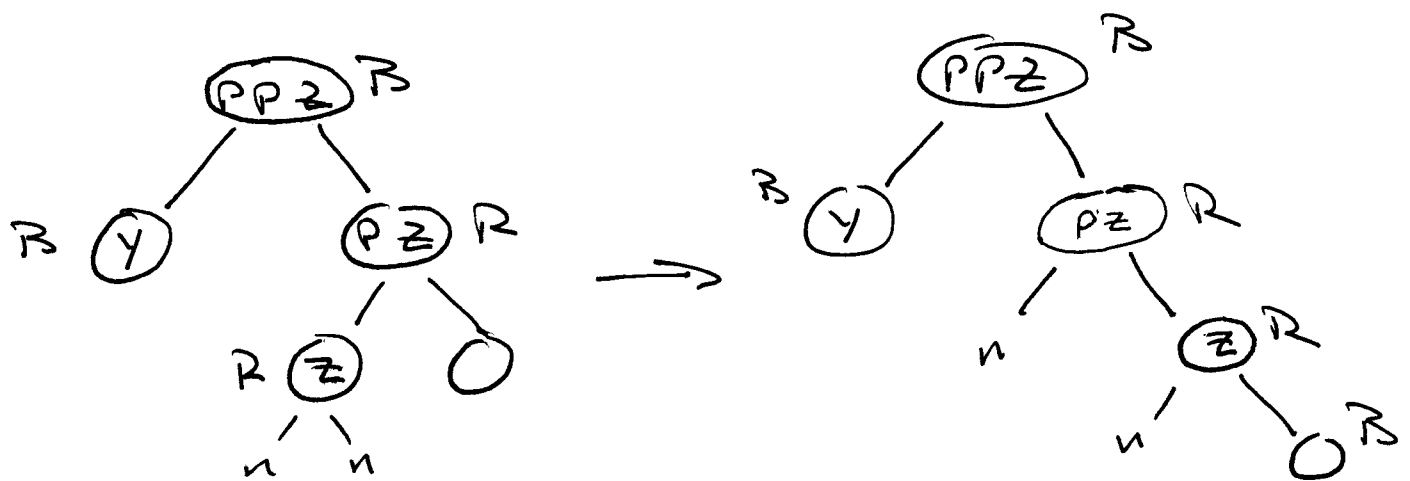
↑ uncle of z

Case 4 : y is Red



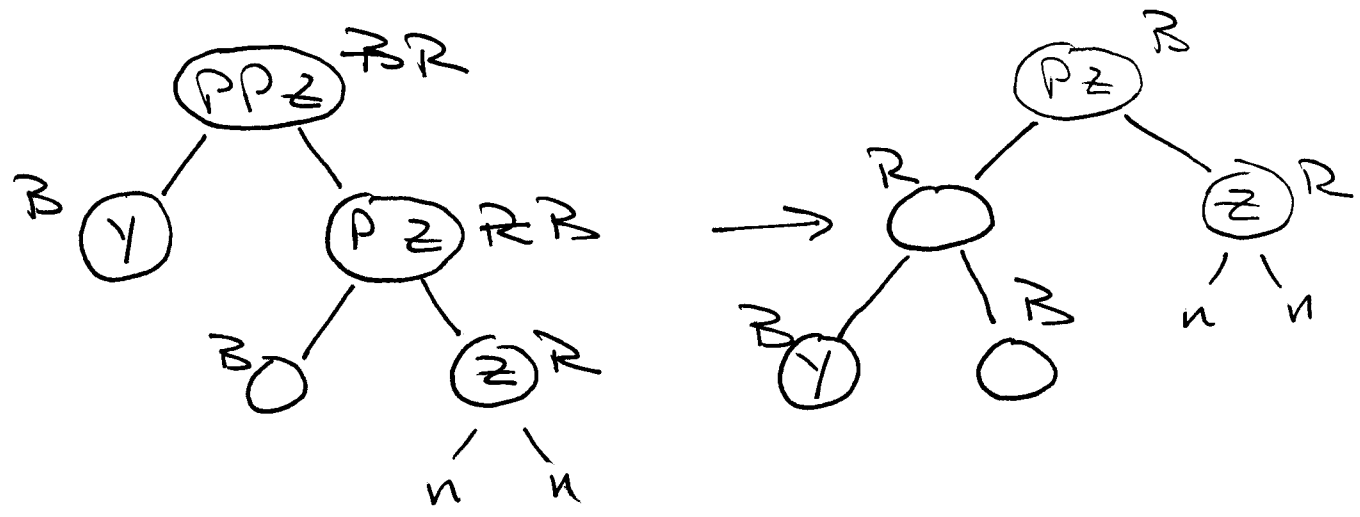
case 5: y is Black and
 $z == z.\text{Parent}.\text{left}$

convert to case 6 by letting
 z climb-up to $z.\text{Parent}$, then
 right rotate about z :



case 6: y is Black, and
 $z == z.\text{Parent}.\text{right}$

we color PPz Red, then LeftRotate
 about PPz ,
 and Pz Black



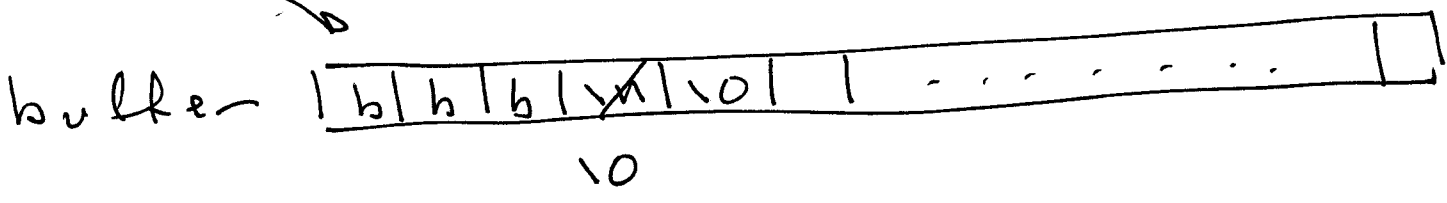
Exercise Insert into an initially empty B+T Keys: 1, 2, 3, 4, 5, 6, 7, 8
 Running B+T insert & Fixup as you go.
 Draw Pictures.

Pa 5: stuff

Input file.

b	b	b	\n		
c	c	c	c	c	\n
a	a	a	\n		

stack



char**

