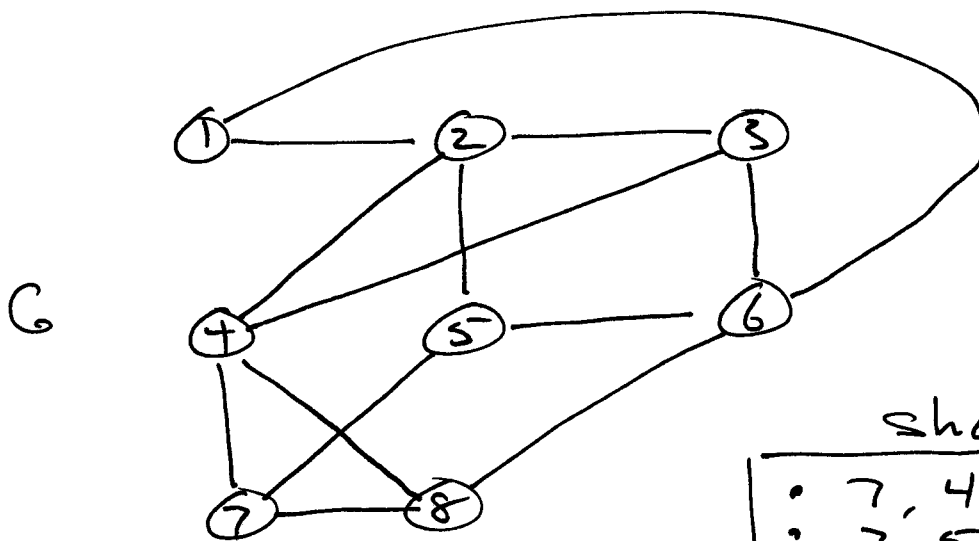


Pa1: ext. 1 day (last)

Pa2: Posted.  $\rightarrow$  BFS

Previous example:



distance

$$g(1, 7) = 3$$

Shortest 7-1 Path

- |              |                   |
|--------------|-------------------|
| • 7, 4, 2, 1 | ← found by<br>BFS |
| • 7, 5, 2, 1 |                   |
| • 7, 8, 6, 1 |                   |

adj[G]: adjacency list representation

- 1 : 2, 6
- 2 : 1, 3, 5
- 3 : 2, 4, 6
- 4 : 2, 3, 7, 8
- 5 : 2, 6, 7
- 6 : 1, 3, 5, 8
- 7 : 4, 5, 8
- 8 : 4, 6, 7

## Breadth First Search (BFS)

- solves SSSP
- uses adj list rep.
- uses vertex attributes:
  - $color[x]$ : white, gray, black
  - $d[x]$ : estimate of  $d(s, x)$
  - $p[x]$ : Predecessor of  $x$  along a shortest  $s-x$  path.
- uses a FIFO queue of vertices:  $Q$

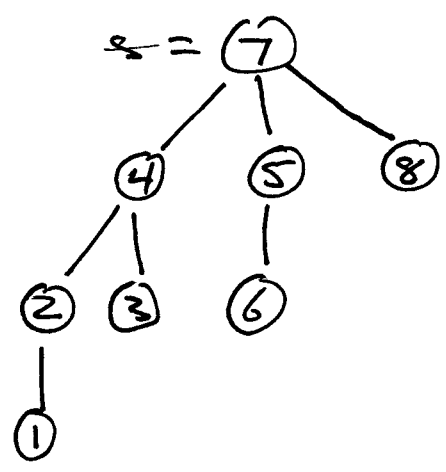
Ex.

|   | adj                                 | color                       | d            | p            |
|---|-------------------------------------|-----------------------------|--------------|--------------|
| 1 | <u>2</u> <u>6</u>                   | <del>w</del> <del>g</del> b | <del>3</del> | <del>2</del> |
| 2 | <u>1</u> <u>3</u> <u>5</u>          | <del>w</del> <del>g</del> b | <del>2</del> | <del>4</del> |
| 3 | <u>2</u> <u>4</u> <u>6</u>          | <del>w</del> <del>g</del> b | <del>2</del> | <del>4</del> |
| 4 | <u>2</u> <u>3</u> <u>7</u> <u>8</u> | <del>w</del> <del>g</del> b | <del>1</del> | <del>7</del> |
| 5 | <u>2</u> <u>6</u> <u>7</u>          | <del>w</del> <del>g</del> b | <del>1</del> | <del>7</del> |
| 6 | <u>1</u> <u>3</u> <u>5</u> <u>8</u> | <del>w</del> <del>g</del> b | <del>2</del> | <del>5</del> |
| 7 | <u>4</u> <u>5</u> <u>8</u>          | g b                         | 0            |              |
| 8 | <u>4</u> <u>6</u> <u>7</u>          | <del>w</del> <del>g</del> b | <del>1</del> | <del>7</del> |

let  $s = 7$

Q: 7 4 5 8 2 3 6 1

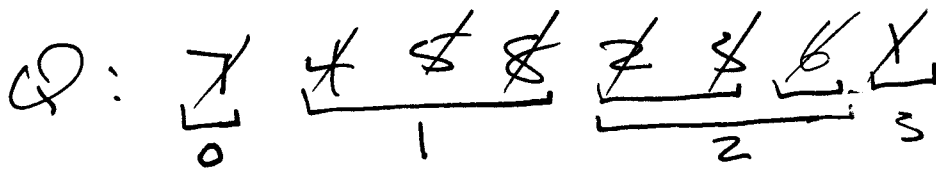
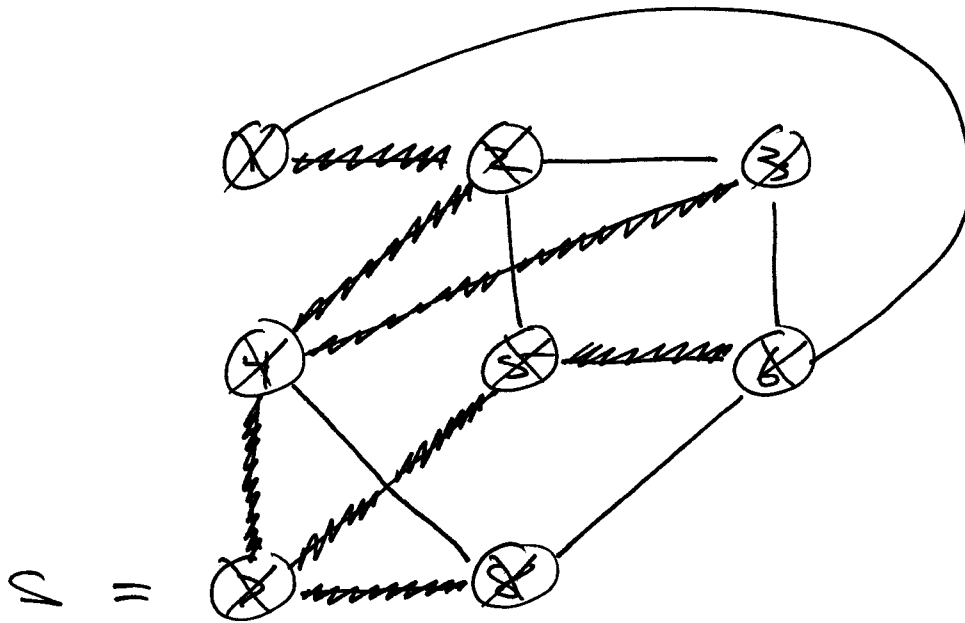
BFS Tree:



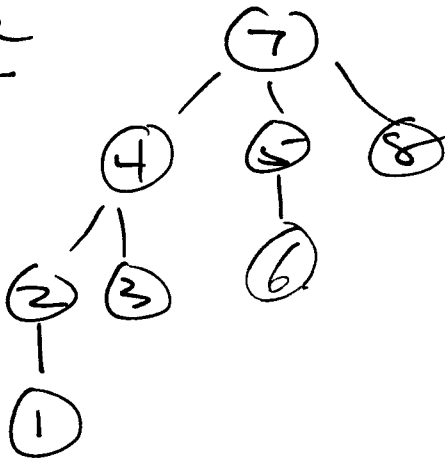
dist = depth

- 0
- 1
- 2
- 3

# Visual approach:



Tree



← Predecessor subgraph.

Defn

$H$  is a subgraph of  $G$  iff

- $H$  is a graph
- $V(H) \subseteq V(G)$
- $E(H) \subseteq E(G)$

Predecessor subgraph of  $G$

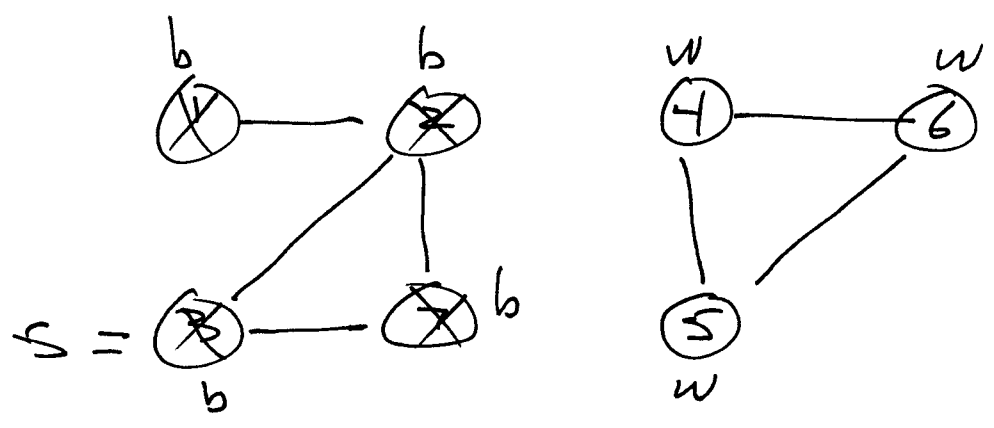
$(V_p, E_p)$  where

$$V_p = \{x \in V(G) \mid p[x] \neq \text{nil}\} \cup \{s\}$$

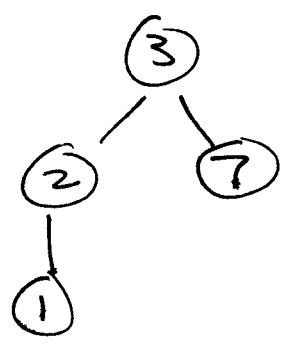
$$E_p = \{ \{x, y\} \mid p[y] = x \}$$

observe:  $|E_p| = |V_p| - 1$  .

Ex. disconnected graph



Q:  $\neq \neq \neq \neq$



dist.  
0  
1  
2

# Directed graphs

$$G = (V, E)$$

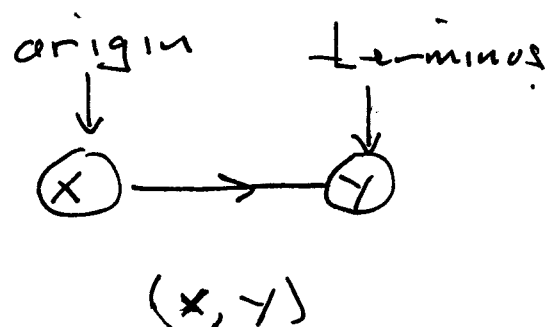
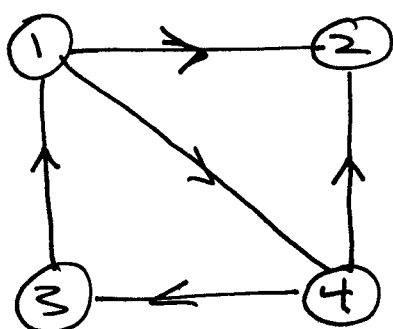
$$V \neq \phi$$

$$E \subseteq V \times V$$

Define  
directed

- walks
- trails
- Paths

Ex.



$$V = \{1, 2, 3, 4\}$$

$$E = \{(1, 2), (1, 4), (3, 1), (4, 2), (4, 3)\}$$

## adj list Representation

|         |   |                                                |
|---------|---|------------------------------------------------|
| 1 : 2 4 | ← | list of termini of edges<br>having 1 as origin |
| 2 :     |   |                                                |
| 3 : 1   |   |                                                |
| 4 : 2 3 |   |                                                |

|   | adj | color | d | p |
|---|-----|-------|---|---|
| 1 | 2 4 |       |   |   |
| 2 |     |       |   |   |
| 3 | 1   |       |   |   |
| 4 | 2 3 |       |   |   |

⋮



# Quiz Practice Problems

```
void swap(List L, int i, int j) {
    int k, x, y, temp;
    if (i > j) { temp = i; i = j; j = temp; }
    moveFront(L);
    for (k = 0; k < i; k++) moveNext(L);
    x = get(L);
    for ( ; k < j; k++) moveNext(L);
    y = get(L);
    set(L, x);
    for ( ; k > i; k--) movePrev(L);
    set(L, y);
}
```

}

```
void reverse(List L, int a, int b) {  
    int i, j, temp;  
    if (a > b) { temp = a; a = b; b = temp; }  
    i = a; j = b;  
    while (i < j) {  
        swap(L, i, j)  
        i++;  
        j--;  
    }  
}
```