cse 101 4-28-26

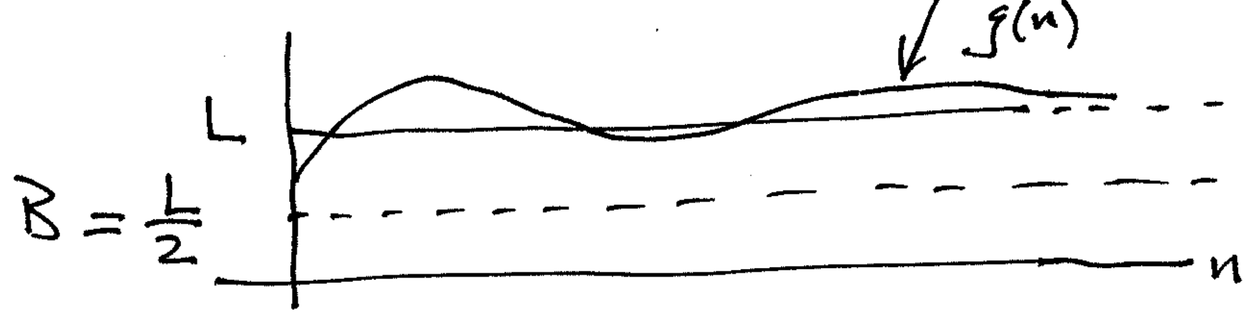
handout: asymptotic growth

note! $\log_b(x) = \frac{\ln(x)}{\ln(b)} = \text{const} \cdot \ln(x)$

Exercises

(1) (a) $\frac{f(n)}{g(n)} \longrightarrow L \in [0, \infty)$

- if $L = 0$, $f = o(g) \Rightarrow f = O(g)$.
- if $0 < L < \infty$, then



$\therefore f = O(g)$

(2) (a) Let $c > 0$. Then

$$cf(n) = O(f(n))$$

why?

$$\frac{c \cdot f(n)}{f(n)} = c \le 2c = B$$

(5) $$\log_b(n) = \frac{\ln(n)}{\ln(b)} = \frac{\ln(a)}{\ln(a)} \cdot \frac{\ln(n)}{\ln(b)}$$

$$= \left(\frac{\ln(a)}{\ln(b)}\right) \cdot \frac{\ln(n)}{\ln(a)} = \text{const} \cdot \log_a(n)$$

$$\therefore \log_b(n) = \Theta(\log_a(n))$$

(6) $f(n) + o(f(n)) = \Theta(f(n))$ ✓

Proof

Let $h(n) = o(f(n))$. Then

$$\frac{h(n)}{f(n)} \longrightarrow 0 \quad \text{as } n \to \infty.$$

So

$$\frac{f(n)+h(n)}{f(n)} = 1 + \underbrace{\frac{h(n)}{f(n)}}_{\downarrow \atop 0} \longrightarrow 1 \in (0,\infty)$$

$\therefore$ by (2)(b) have

$$f(n) + h(n) = \Theta(f(n))$$

Ex. Selection Sort

array of ints.

SelectionSort(A)

```
n = length(A)
for i = 1 to n-1
    imin = i
    for j = i+1 to n
        if A[j] < A[imin]
            imin = j
    A[i] <-> A[imin]    // swap
```

Basic op.: $\boxed{A[j] < A[imin]}$

Picture:

$$\underbrace{A_1 \cdots A_{i-1}}_{\text{sorted}} \underbrace{A_i \; A_{i+1} \cdots A_n}_{\text{unsorted}}$$

- sorted
- $(i-1)$ smallest elements

unsorted

Basic operation: array comp. line #5

$$\# \text{comp} = (n-1) + (n-2) + \cdots + 1 = \frac{n(n-1)}{2}$$

$$= \frac{1}{2} n^2 - \frac{1}{2} n = \Theta(n^2)$$

- BFS(G, s)

  Basic op: assignment

  instance size = n+m

  n = #vert, m = #edges.

Initialization cost = $\Theta(n)$

Queue ops. cost = $\Theta(n)$ } worst case

Processing adj lists = $\Theta(m)$

total cost = $\Theta(n+m)$