cse 101-02 6-8-23

- Final exam : Thur 6/15 12:00-2:00 PM
- SETs : closes Sun. 11:59 PM
- Pa8 : ext. 1 last day ⇒ Sat. 10:00 PM

P.Q. OPs   runtimes

HeapMaximum : $\Theta(1)$

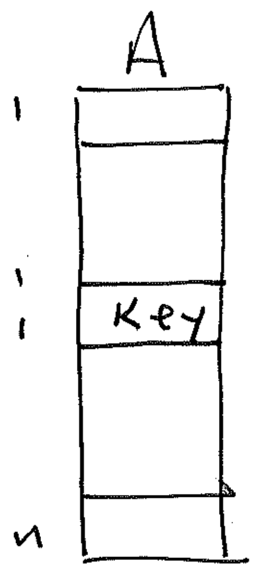
HeapDeleteMax : $\Theta(\log n)$

HeapExtractMax : $\Theta(\log n)$

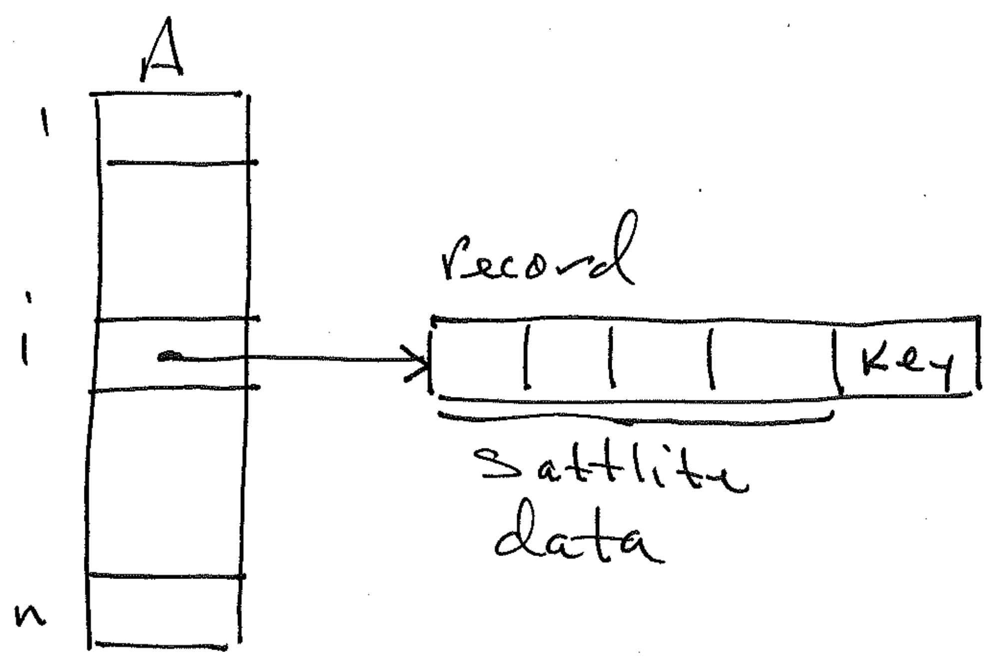
HeapIncreaseKey : $\Theta(\log n)$

HeapInsert : $\Theta(\log n)$

OUR Picture



General Picture



Exercise: re-write algorithms in General Picture.

SSSP in Weighted Graph

ch. 24 in CLRS

Defn

- a weighted graph is a graph $G=(V,E)$ with a fcn

$$w : E \longrightarrow \mathbb{R}$$

- Path weight of $P: x=v_0, v_1, \ldots, v_k = y$

$$w(P) = \sum_{i=1}^{k} w(v_{i-1}, v_k)$$

- shortest path weight : $\delta(x,y)$

$$\delta(x,y) = \begin{cases} \min\{w(P) \mid P \text{ is an } x\text{-}y \text{ Path}\} \\ \infty \text{ if no } x\text{-}y \text{ Path exists} \end{cases}$$

- SSSP Problem:

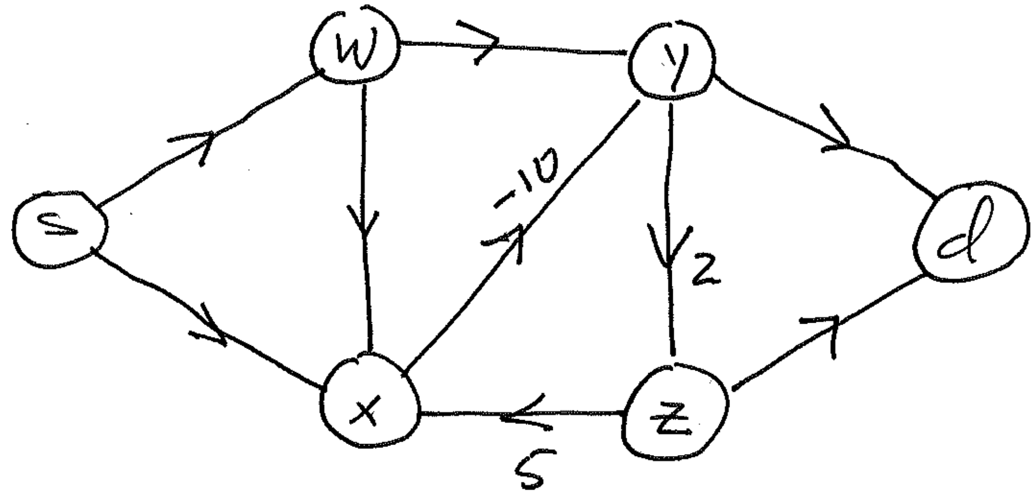
Given a graph $G=(V,E)$ and a source $s \in V$:

(1) determine $\delta(s,x)$ for all $x \in V$.

(2) for those x s.t. $\delta(s,x) < \infty$, find p, an s-x path, s.t.

$$w(p) = \delta(s,x).$$

Ex.



cycle: x, y, z, x weight = −3

Dijkstra: Outlaws negative weight edges.

Infrastructure for Dijkstra:

for each $x \in V(G)$

- $p[x]$: Parent of x, encodes a shortest path tree
- $d[x]$: estimate of $\delta(s,x)$

Predecessor subgraph: $G_p = (V_p, E_p)$

$$V_p = \{ x \in V(G) \mid p[x] \neq nil \} \cup \{ s \}$$

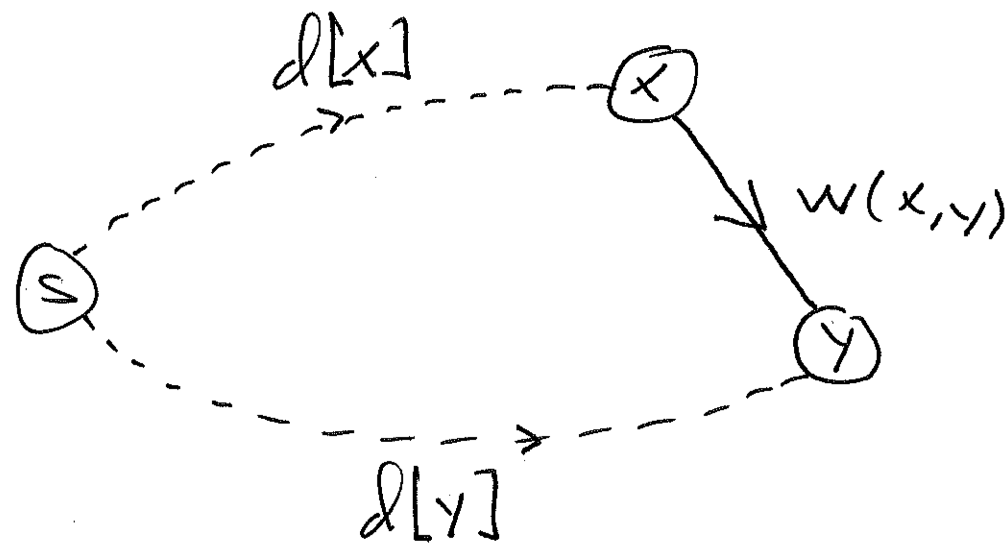
$$E_p = \{ (p[x], x) \mid p[x] \neq nil \}$$

Use! PrintPath(G, s, x)

helper functions:

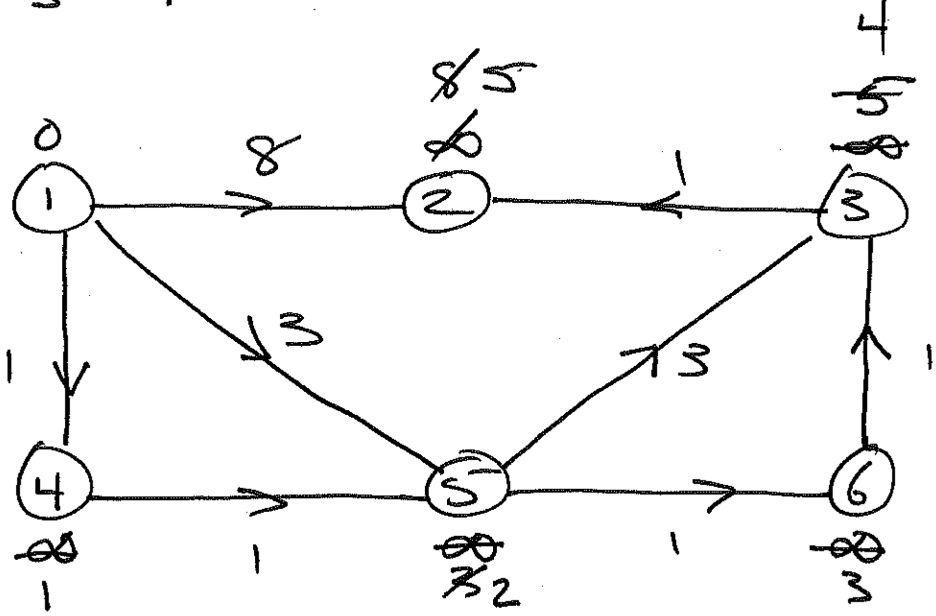
· Initialize(G, s)

· Relax(x, y) pre: $(x, y) \in E(G)$



Pseudo-code for Dijkstra.

Ex. $s = 1$

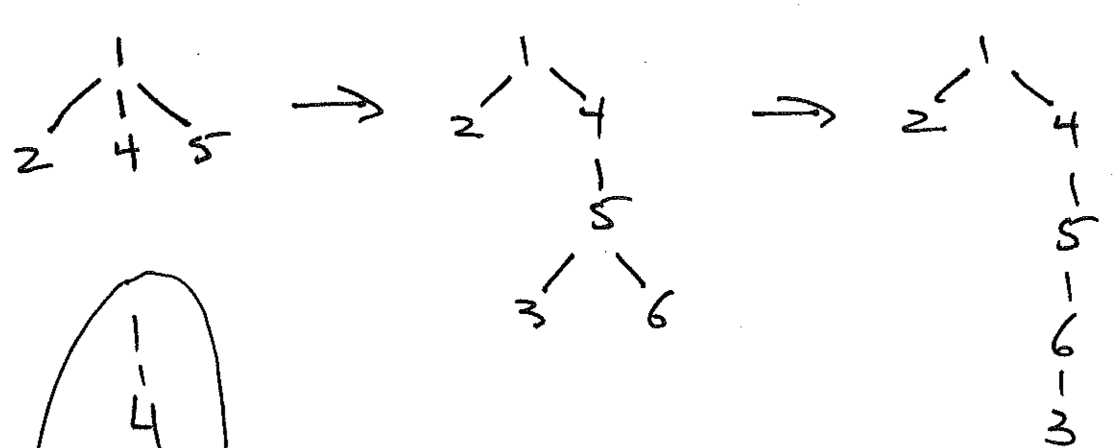


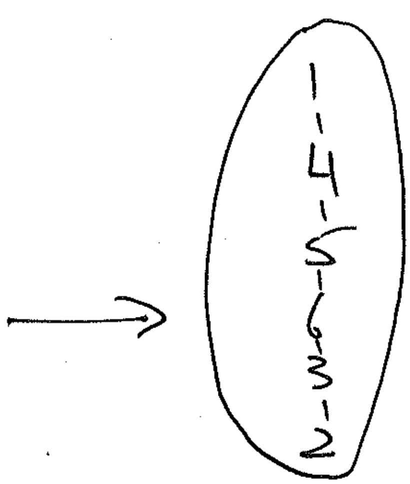
Remove order

1
4
5
6
3
2

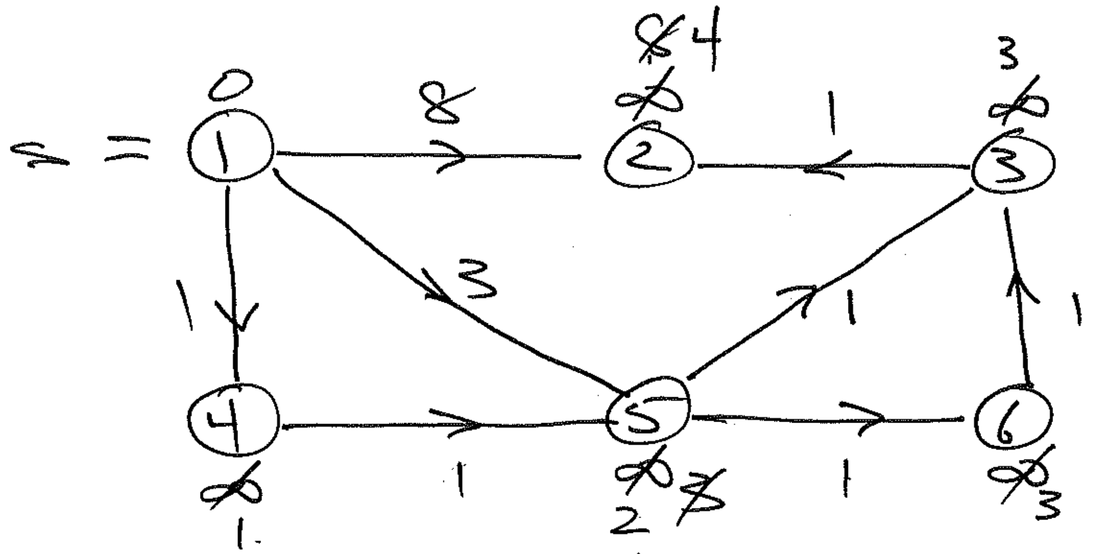
PQ :	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d :	0	~~8~~5	~~5~~4	1	~~3~~2	3
p :	n	~~1~~3	~~5~~6	1	~~1~~4	5

Tree





Ex. Rule: if two vertices have identical minimum keys (d-values) extract the one with lowest label.



Rem. Order
1
4
5
3
6
2

P.Q.	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d	0	~~8~~ 4	3	1	~~3~~ 2	3
p	n	~~1~~ 3	5	1	~~1~~ 4	5

Tree