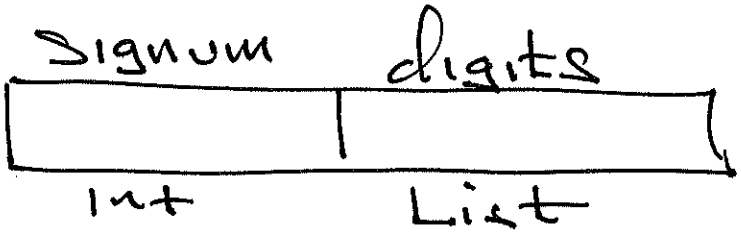
CSE 101-02 5-16-23

Pa6: BigInteger



+1, -1, 0 (. , . , . , . , .)

sign magnitude

Ex. addition $b=100$, $p=2$

$$\begin{array}{r} {\scriptstyle 1\quad\ 1\quad\ 1\quad\ 0} \\ (88\ \ 21\ \ 33) = 882{,}133 \\ (65\ \ 91\ \ 79) = 659{,}179 \\ \hline {\scriptstyle 1\quad\ 1\quad\ 1\quad\ 0} \\ (153\ \ 112\ \ 112) \\ -100\ \ -100\ \ -100 \\ \hline (1,\ 54,\ 13,\ 12) \end{array} \quad \begin{array}{l} \\ \\ \\ \\ \text{vector sum} \\ \\ \text{normalize} \end{array}$$

$1,541,312$

another way

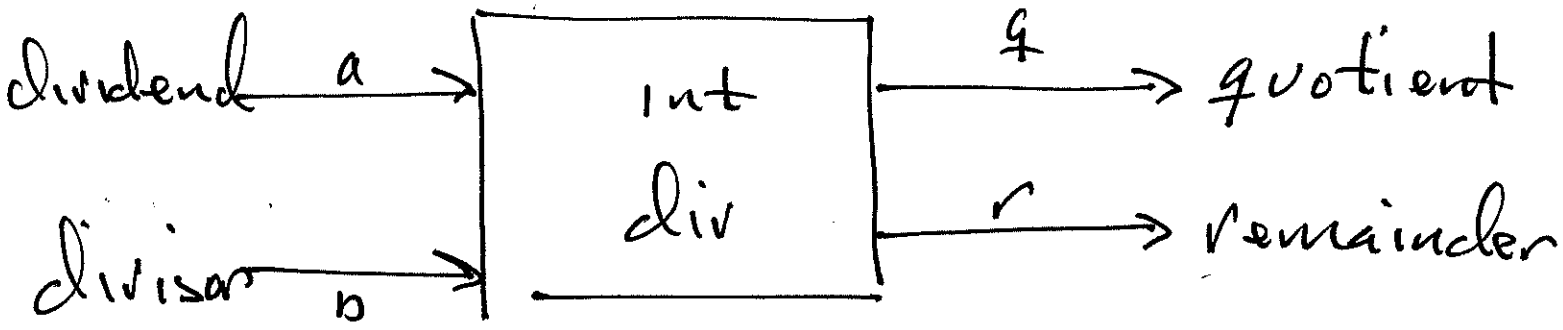
Ex. Subtraction $b = 100$, $p = 2$

$$\begin{array}{r} (88\ \ 21\ \ 33) = 882,133 \\ (65\ \ 91\ \ 79) = -659,179 \\ \hline (22\ \ 29\ \ 54) \qquad 222,954 \end{array}$$

another way:

$$\begin{array}{l} (88\ \ 21\ \ 33) \\ (65\ \ 91\ \ 79) \\ \hline (23\ \ -70\ \ -46) \quad \text{vector diff.} \\ \qquad\ 100\ \ 100 \\ (22\ \ 29\ \ 54) \quad \text{normalize} \end{array}$$

Integer division !



2 Properties:

$$\begin{cases} a = q\,b + r \\ 0 \le r < b \end{cases}$$

$$\left.\begin{aligned} q &= a/b \\ r &= a\,\%\,b \end{aligned}\right\} \text{ in C++}$$

Ex Subtraction $b=100$, $p=2$

$(65 \quad 91 \quad 79)$

$(88 \quad 21 \quad 33)$

$(-23 \quad 70 \quad 46)$ vector diff.

↑ leftmost digit is negative

$-1 \quad -1 \quad 0$

$-1 \quad (23 \quad -70 \quad -46)$

-71

$100 \quad 100$

$(22 \quad 29 \quad 54)$ normalize

Pull out -1, change signs in list, normalize again, return -1.

Ex. normalize: $b = 10$

$$\begin{matrix} -9 & +1 & +6 & -5 & -50 & 0 \end{matrix}$$
$$(1, -90, 9, 73, 0, -500)$$

$$-89 \quad 15 \quad 68 \quad -50$$

$$+90 \quad -10 \quad -60 \quad +50 \quad +500$$

$$(-8, 1, 5, 8, 0, 0)$$

$$\begin{matrix} -1 & -1 & -1 & 0 & 0 & 0 \end{matrix}$$
$$\boxed{-1} \quad (8, -1, -5, -8, 0, 0)$$

$$-2 \quad -6$$

$$10 \quad 10 \quad 10$$

$$(7, 8 \quad 4, 2, 0, 0)$$

↓ return

check: $1 \cdot 10^5 + (-90) \cdot 10^4 + 9 \cdot 10^3 + 73 \cdot 10^2 + 0 \cdot 10^1 + (-500) \cdot 1$

$$= -784200$$

Ex. multiplication: $b = 10$, $p = 1$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 123 \\ \times\ 456 \\ \hline 738 \\ 6150 \\ 49200 \\ \hline 56088 \end{array}$$

(1 2 3) ← n digits

(4 5 6) ← m digits

			shift
* ~~empty~~	( )		
	(6 12 18)	scalar mult "add"	0
*	(7 3 8)	normalize	
	(5 10 15 0)	scalar mult	1
	(5 17 18 8)	add	
*	(6 8 8 8)	normalize	
	(4 8 12 0 0)	scalar mult	2
	(4 14 20 8 8)	add	
*	(5 6 0 8 8)	normalize	

Runtime:

digit multiplications $= n \cdot m$

digit additions $= 2m \cdot n + m^2$

runtime $= \Theta(nm)$

Ex. $b = 10, \quad p = 1$

$$\begin{array}{r} 999 \\ 999 \\ \hline 998001 \end{array}$$

(9 9 9)

(9 9 9) shift

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empty	( )		
	(81 81 81)	s.m	0
	(81 81 81)	add	
	(8 9 9 1)	norm	
	(81 81 81 0)	s.m.	1
	(89 90 90 1)	add	
	(9 8 9 0 1)	norm	
	(81 81 81 0 0)	s.m	2
	(90 89 90 0 1)	add	
	(9 9 8 0 0 1)	norm	

Exercise

show that in base b

$$(\underbrace{b-1, b-1, b-1, \ldots, b-1}_{n})^2$$

$$= (\underbrace{b-1, b-1, \ldots b-1}_{n-1}, b-2, \underbrace{0, \ldots 0}_{n-1}, 1)$$

e.g. in $b = 10$:

$$99999^2$$

$$= 9999800001 \quad \text{(check)}$$