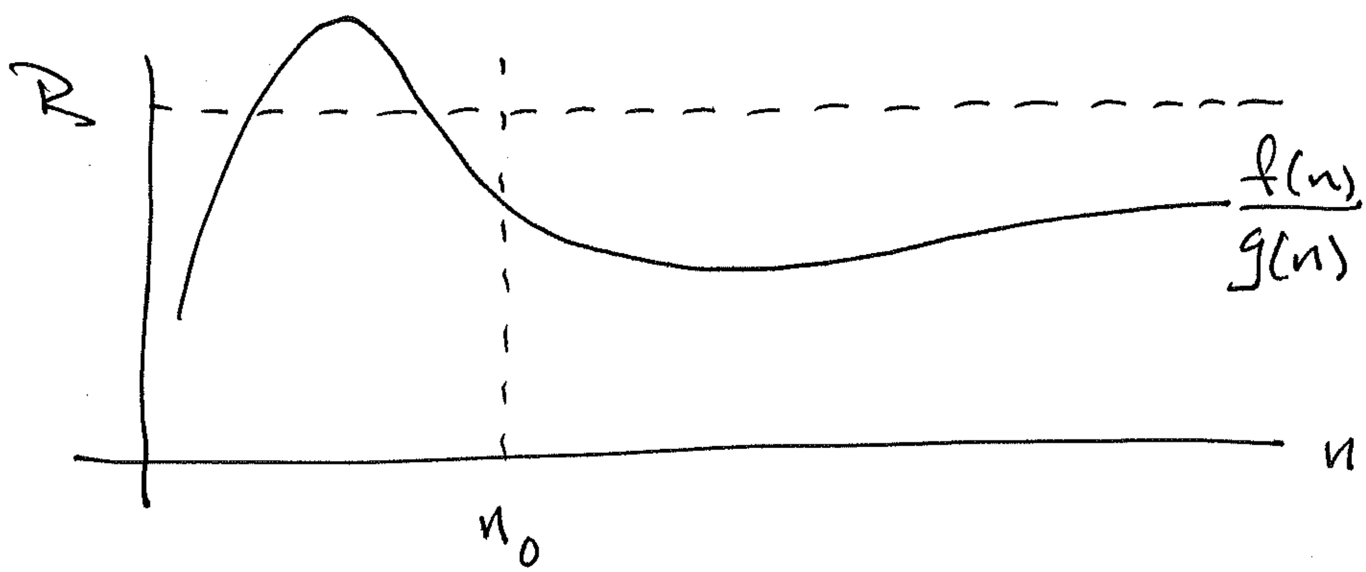
CSE 101-02 4-27-23

Recall defn of $f(n) = O(g(n))$.

<u>Picture</u>:



<u>Ex.</u> let $f(n) = 2n+5$, $g(n) = n$. Then

$$\frac{f(n)}{g(n)} = 2 + \frac{5}{n} \leq 3$$

for all $n \geq 5$. $\therefore B = 3$, $n_0 = 5$ work.

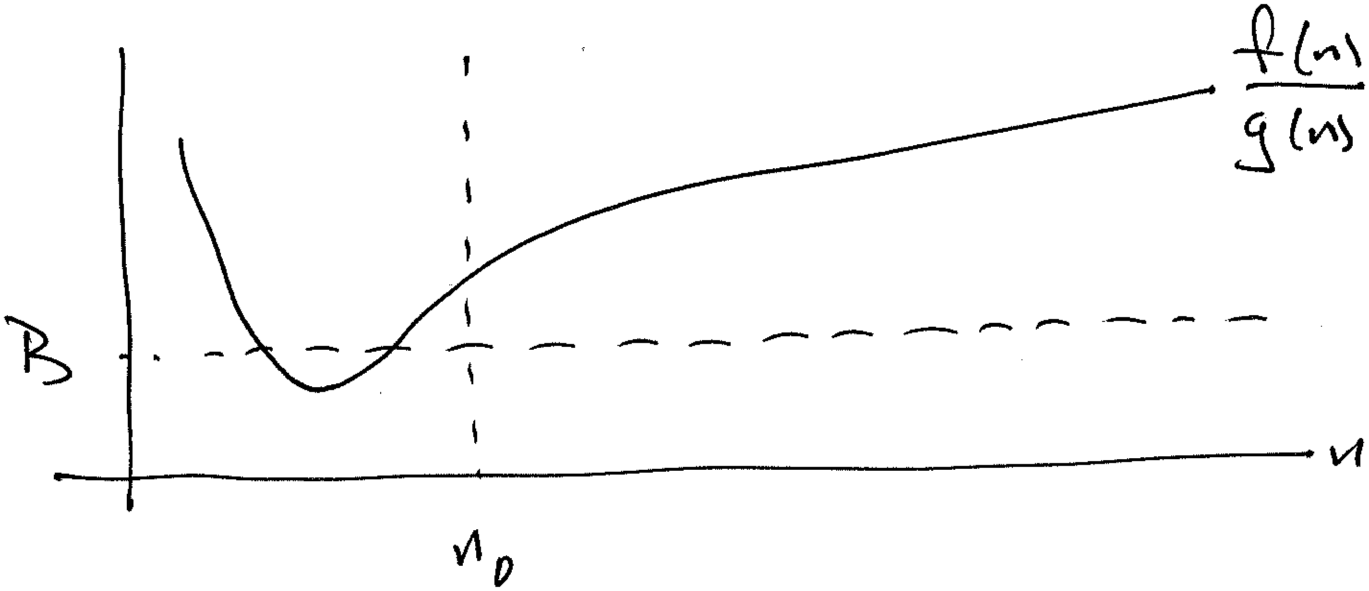
Defn

Write $f(n) = \Omega(g(n))$ iff there exist $B > 0$, $n_0 > 0$ s.t.

$$\frac{f(n)}{g(n)} \geq B$$

for all $n \geq n_0$. Say $g(n)$ is an asymptotic lower bound to $f(n)$.

Picture:



Ex. $f(n) = 6n^3 + 4n$, $g(n) = 2n^2$.

Then

$$\frac{f(n)}{g(n)} = 3n + \frac{2}{n} \geq 7$$

for all $n \geq 2$.

Fact: If $P(n)$, $Q(n)$ are Polynomials, then

- $P(n) = O(Q(n))$ iff $\deg(P(n)) \leq \deg(Q(n))$
- $P(n) = \Omega(Q(n))$ iff $\deg(P(n)) \geq \deg(Q(n))$.

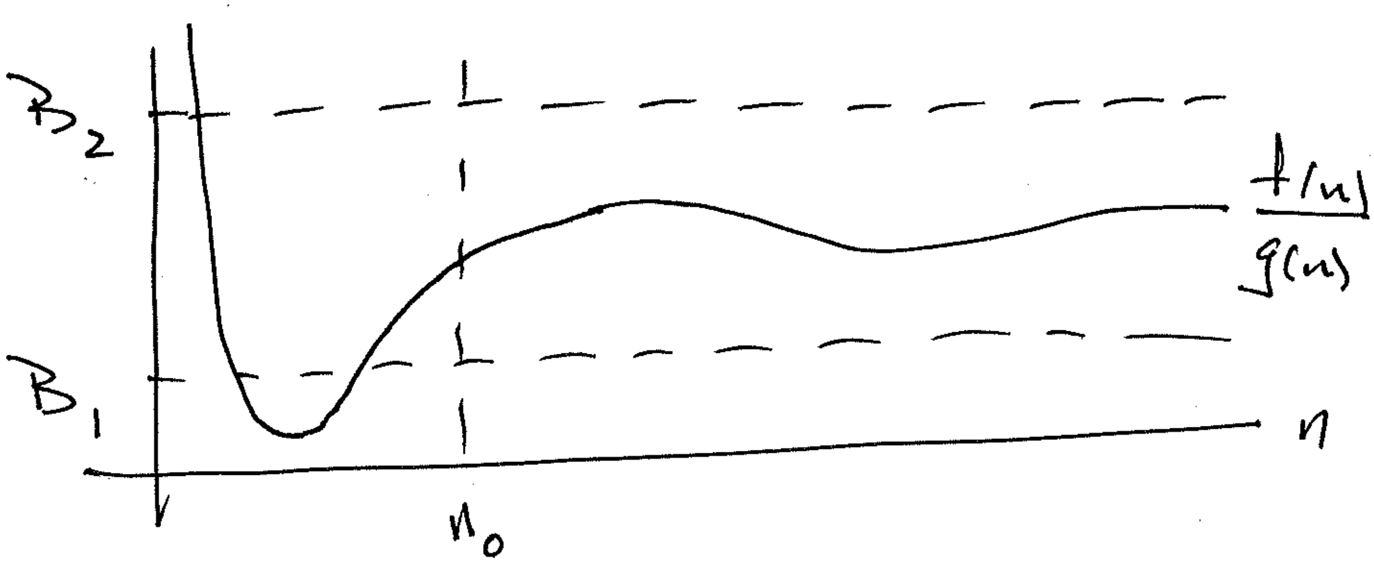
Defn

write $f(n) = \Theta(g(n))$ iff there exist: $B_1 > 0$, $B_2 > 0$, $n_0 > 0$ s.t.

$$B_1 \le \frac{f(n)}{g(n)} \le B_2$$

for all $n \ge n_0$. Say $g(n)$ is a tight asymptotic bound for $f(n)$.

Picture:



<u>Ex</u> $f(n) = 5n^2 + 11n - 24$, $g(n) = n^2$.

check that

$$4 \leq \frac{5n^2 + 11n - 24}{n^2} \leq 6$$

for all $n \geq 8$. (so $B_1 = 4$, $B_2 = 6$, $n_0 = 8$)

i.e.

$$4 \leq 5 + \frac{11}{n} - \frac{24}{n^2} \leq 6$$

<u>Fact</u>! if $P(n)$, $Q(n)$ Polynomials, then

$$P(n) = \Theta(Q(n)) \text{ iff } \deg(P(n)) = \deg(Q(n))$$